



A Technical Manual of the Geocentric Datum Brunei Darussalam 2009 (GDBD2009)

Version 1.1

**Survey Department
Ministry of Development
Brunei Darussalam**

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CHAPTER 1

INTRODUCTION

1.1 Preamble

Brunei Darussalam is situated on the north-west of the island of Borneo, between east longitudes $114^{\circ} 04'$ and $114^{\circ} 23'$ and north latitudes of $4^{\circ} 00'$ and $5^{\circ} 05'$. It has a total area of 5,765-sq-km. with a coastline of about 161 km along the South China Sea. It is bounded on the North by the South China Sea and on all the other sides by Malaysian State of Sarawak.

In 1847, the sultan concluded a treaty with Great Britain and in 1888 Brunei Darussalam officially became a British protectorate. In 1906, the Residential System was established in Brunei Darussalam. On January 1, 1984 Brunei Darussalam resumed full independence and the Sultan took office as Prime Minister, Finance Minister and Home Affairs Minister, presiding over a cabinet of six.

Survey department in the development of modern history has already achieved lots of improvement. As one of the agency under the government of Kebawah Duli Yang Maha Mulia Paduka Seri Baginda Sultan dan Yang Dipertuan Negara Brunei Darussalam, the Ministry of Development, Survey Department has done its role in response to our country needs in related to matters of land surveying, preparation and production of maps and consistency and updating of Land Information Technology toward country development and projects under the Country Development Plan.

1.2 Existing Geodetic Networks

1.2.1 BRUNEI TRIANGULATION NETWORK

Prior to 1948 triangulation of Eastern Sarawak, Brunei and Labuan was part of primary triangulation of British Borneo. The original survey of Brunei was

conducted in the period 1934 to 1937, (Bridges 1937). The survey was motivated by the need to provide geodetic infrastructure for development, (Bridges 1937). In 1947 it became evident that the artificial division of Sarawak into two sections (each with a different origin) would eventually lead to confusion and the Directorate of Colonial Surveys, with the agreement of the Survey Departments concerned, undertook the task of readjusting the whole of the Primary Triangulation of Borneo from the original observations. From the original observations, preliminary figural adjustments were made, the least-square method of angular adjustment being used. The chain was split up into separate figures. This adjustment proved that the original angular observations were uniformly excellent. The base and azimuth misclosures were also found to be small and these were then eliminated, Laplace azimuth conditions being used at the two points where reliable longitudes had been observed.

The 1968 adjustment for the states of East Malaysia, Sabah and Sarawak, known as the BT68 adjustment, (Abu, 1998) was not adopted in Brunei. This adjustment was performed with Bukit Timbalai as the origin. The coordinates of Bukit Timbalai were consistent with the 1948 adjustment and not those of the 1937 adjustment. The semi-major axis of the ellipsoid was as used in the 1948 solution.

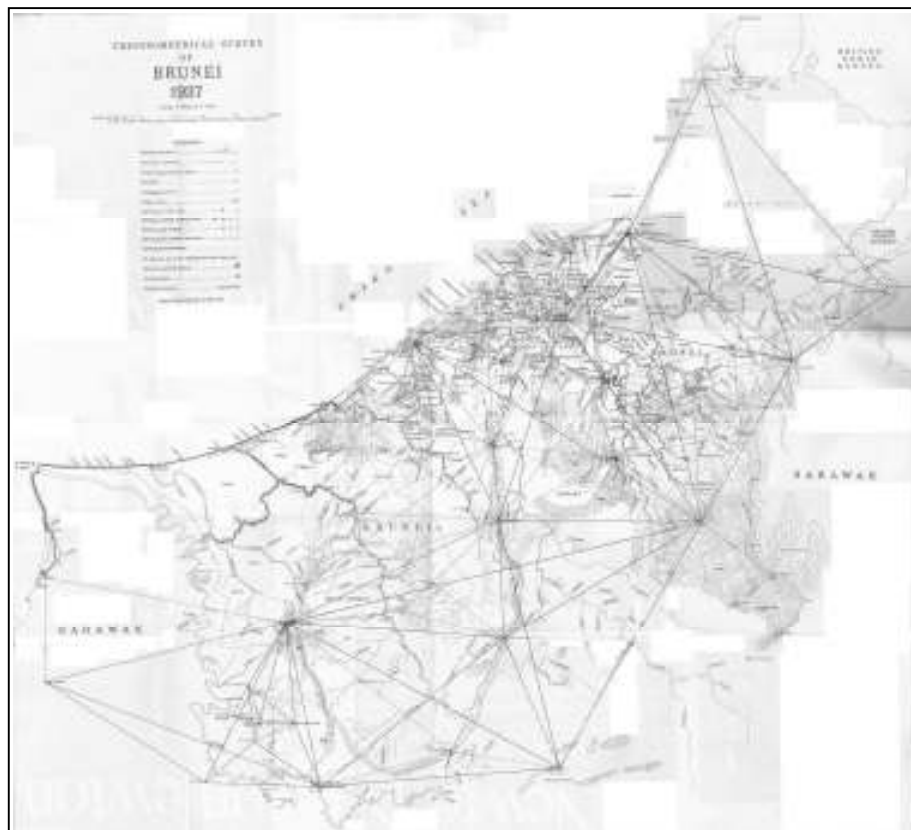


Figure 1.1: Triangulation of Brunei Darussalam

1.2.2 GLOBAL POSITIONING SYSTEM (GPS) NETWORK

A GPS network was established in Brunei Darussalam using Wild Macrometers WM101GPS receivers 1992 (Morgan, 2004). These were quickly replaced with Wild system 200 receivers in 1994 and a major campaign covering all of the Districts of Brunei was undertaken in 1994 and 1995 (Morgan, 2004).

Between 2002 and 2003, a GPS campaign was carried out on seventeen (17) stations which include primary and secondary triangulation stations and new GPS monument. The main objectives are to establish a new GPS network, analyze the existing geodetic network and obtain transformation parameters between WGS84 of GPS and BT48.

CHAPTER 2

REALIZATION OF GDBD2009

2.1 INTRODUCTION

Survey Department Brunei Darussalam is responsible for the establishment and maintenance of horizontal and vertical control points for geodetic applications. GNSS had been introduced at Survey Department in early 1990s. To date, it has been used in the establishment of GNSS networks in the Brunei.

The realization of the Geocentric Datum for Brunei Darussalam (GDBD) is based on a network of permanent GPS tracking stations in Brunei, which fits into the global ITRF geodetic framework. Currently, Brunei's GNSS permanent stations consists of three (3) existing active permanent GNSS tracking stations and added up with another five (5) new stations in 2009.

The adoption of a geocentric datum is inevitable considering that satellite positioning systems would have widespread use in this millennium and the positions referenced to the existing datum would not be compatible with such satellite derived positions. The adoption of a global geocentric datum would allow for a single standard for the acquisition, storage and the use of geographic data, thus ensuring compatibility across various GIS applications.

The International Earth Rotation Services (IERS) maintains this present day terrestrial reference system through an International Terrestrial Reference Frame (ITRF), which is defined by adopting the geocentric Cartesian coordinates and velocities of global tracking stations derived from the analysis of VLBI, SLR, and GPS data. The implementation of geocentric datum for Brunei Darussalam will require the connection to such reference frame (ITRF) through network of permanent GNSS stations manage by The International GNSS Service (IGS).

2.2 GNSS PERMANENT NETWORK

GNSS permanent network in Brunei is the latest venture of Survey Department in providing 24 hours GNSS data for GNSS users in Brunei. Currently, Survey Department has eight (8) Continuous Operation Reference Stations (CORS) for the network covering the whole Brunei Darussalam. Each reference station is equipped with a Trimble NetR8 GPS receiver, antenna, power supply and modem to communicate with the control centre via communication infrastructure.

Real Time Kinematic (RTK) survey method is the latest innovation of relative positioning whereby two receivers are linked by radios simultaneously in collecting observations. RTK is now widely used for surveying and other precise positioning applications.

The new generation of RTK known as “Virtual Reference Station” is based on having a network of GPS reference stations continuously connected via telecommunication network to the control centre. A computer at the control centre continuously gathers the information from all receivers, and creates a living database of Regional Area Corrections. With VRS system, one can establish a virtual reference station at any point and broadcast the data to the roving receivers.

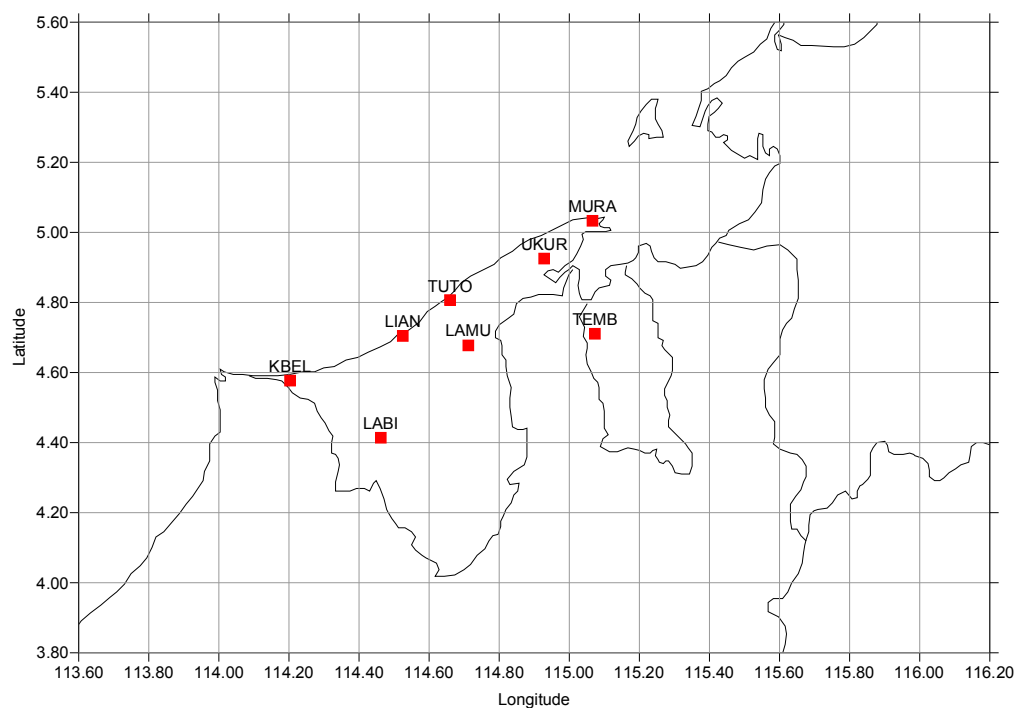


Figure 2.1: RTK Stations

In order to take full advantage of the real-time capabilities of the VRS system, Survey Department Brunei Darussalam has established a network of permanently running GNSS base stations, at spacing about 30 km, feeding GNSS data to a processing centre via a computer network. A central facility was setup to model the spatial errors that limit GNSS accuracy through a network solution and generate corrections for roving receivers to be positioned anywhere inside the network with an accuracy better than a few centimetres to a few decimetres in real time. At the same time, a web site was made available to download GNSS data for post-processing solutions.

2.2.1 RTK Network Design

Modern access technologies are faced with several conflicting goals. It is desirable to connect multiple hosts at a remote site through the same customer premise access device. It is also a goal to provide access control and billing functionality in a manner similar to dial-up services using PPP. In many access technologies, the most cost effective method to attach multiple hosts to the customer premise access device is via Ethernet. In addition, it is desirable to keep the cost of this device as low as possible while requiring little or no configuration.

The primary goals in designing the system were for efficient and easy operations facilitate user understanding of the network and for expandability. Some empirical numerical limits have been suggested for IP addressing and efficient operation of the system. RTK network is using ADSL (Asynchronous Digital Subscriber Line) for the connectivity. Before the gateway will pass any data between the LAN (Local Area Network) interface and the WAN (Wide Area Network) interface, the WAN side of the DSL modem must be configured. Depending upon the DSL service provider or the ISP (Internet Service Provider), it will need these settings information outlined below before configuring the WAN properly.

PPPoE (PPP over Ethernet) provides the ability to connect a network of hosts over a simple bridging access device to a remote Access Concentrator. With this model, each host utilizes its own PPP stack and the user is presented with a familiar user interface. Access control, billing and type of service can be done on a per-user, rather than a per-site, basis.

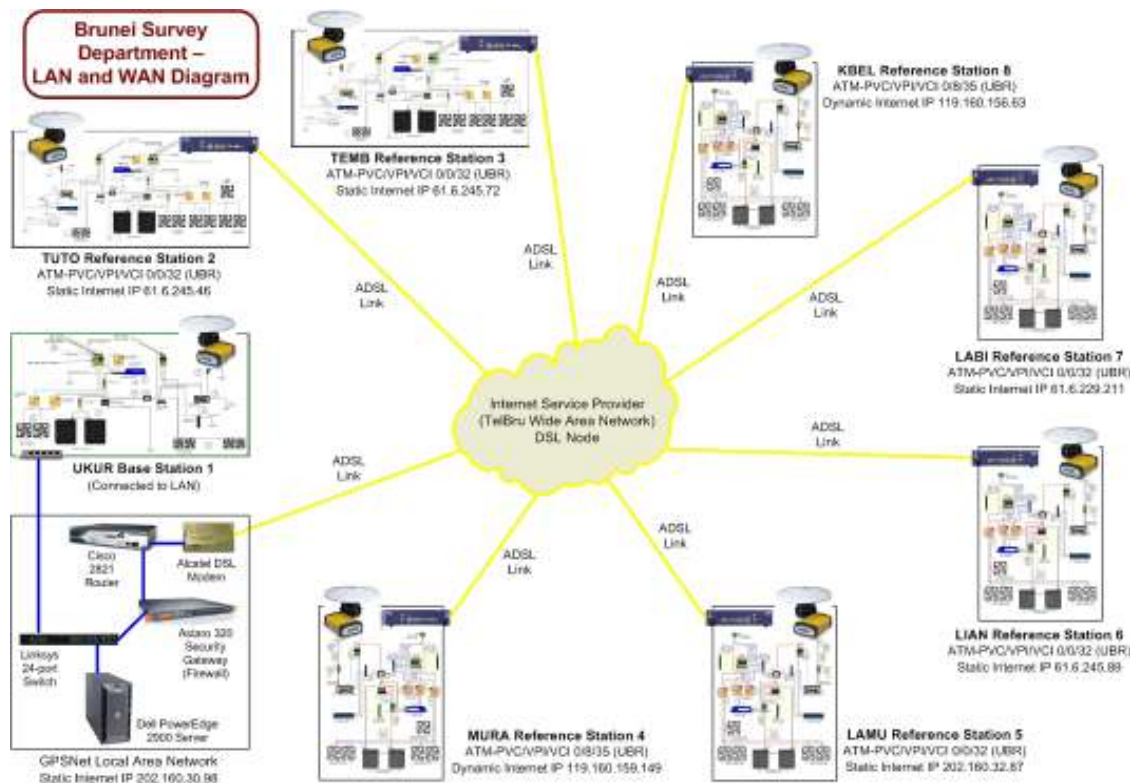


Figure 2.2: RTK Network Configuration and Communication Links

2.2.2 Hardware Configuration for Remote Stations

Connected via ADSL lines, all RTK stations are in direct link-up to the Processing Centre. All functions of the RTK stations such as receiver control, tracking schedules, data acquisition and uploading times are accomplished by the software residing control center. The system is also capable of alerting and engaging onsite personnel of any abnormal conditions occurring at each RTK station such as power failure, system breakdown etc.

At the heart of the system are the GNSS receivers, which are dual frequency Trimble NetR8 models. The receivers have 72 channels, which are set to record GNSS signals at 1 seconds recording interval.

Brunei Survey Department – Enclosure Wiring Diagram

Legend:

Violet/Red = Power Line (+)
Blue/Brown/Black = Neutral (-)
Yellow-Green = Earth-Ground
Bare/Black-White Stripe = Positive (+)
All White – This Color = Negative (-)

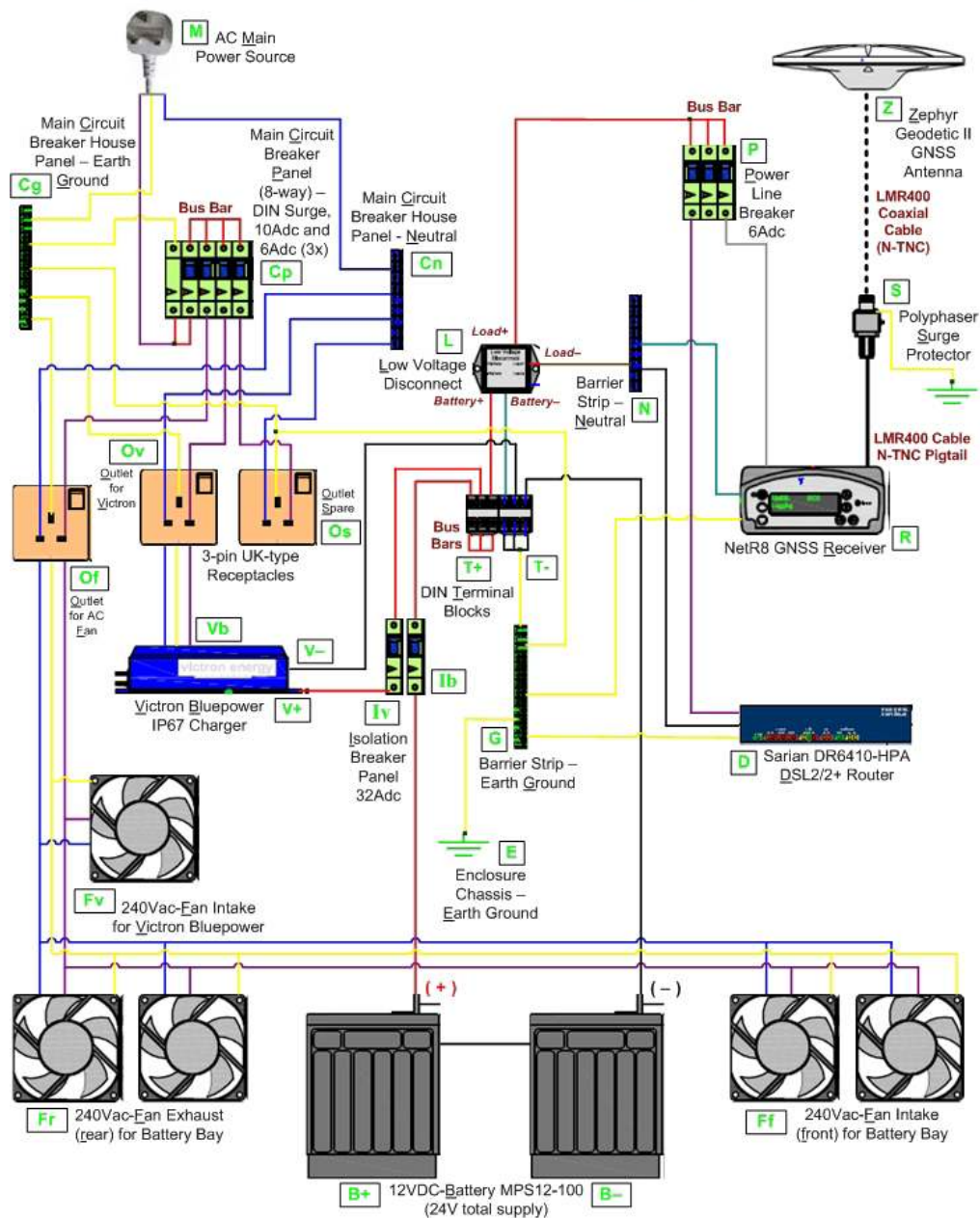


Figure 2.3: Remote Stations Configuration



Figure 2.4 KBEL Reference Station
(Balai Bomba Kuala Belait)



Figure 2.5 LABI Reference Station
(Kawasan Bangunan Pejabat Daerah, Mukim Labi)



Figure 2.6 MURA Reference Station
(Flat kediaman Tentera Laut, Muara)



Figure 2.7 LAMU Reference Station
(Balai Bomba Lamunin)



Figure 2.8 LIAN Reference Station
(Balai Bomba Sungai Liang)



Figure 2.9 TEMB Reference Station
(Flat Kediaman Kementerian Pendidikan, Bangar Temburong)



Figure 2.10: TUTO Reference Station
(Jabatan Daerah Tutong)



Figure 2.11: UKUR Reference Station
(Jabatan Ukur)

2.2.3 Products

RTK system provides the following levels of GNSS correction and data:

a) VRS Correction

Within the limits of RTK network, the system provides Real Time Kinematic Network GNSS corrections with accuracies of 1 to 3 cm horizontally and 3 to 6 cm vertically. Distance dependent errors are considerably minimised with utilisation of the RTK network, thereby achieving increased accuracy and reliability. The above stated accuracy is still achievable within a distance of 30 km away from the dense network.

b) Single Base Real-Time Correction

This correction is provided for area within 30 km from the RTK single reference station with an accuracy of 1 to 3 cm horizontally and 3 to 6 cm vertically.

c) Network Base DGPS Correction

This correction provides better than 50 cm accuracy for the whole of Brunei Darussalam.

d) Virtual RINEX Data

The RTK system provides data for post-processing of static survey sessions, with accuracies of 1 to 3 cm horizontally and 3 to 6 cm vertically. The data is provided in the standardised RINEX Version 2.11 format and is available via our password protected internet website. Data can be downloaded at any interval ranging from 0.1 s to 60 s specified on the web.

2.2.4 Applications

a) Single receiver positioning

In this concept, a user can carry out a survey with only one receiver for the purpose of survey and mapping work such as Cadastral Survey, GIS systems Integration, Map completion etc. Here, the user can avoid the setting-up of a base station, thus saving time, money and energy.

b) Deformation Monitoring

Deformation monitoring such as high rise building, bridge and Dams will contribute to a better maintenance of national infrastructure and land use.

c) Contribution to Global Reference Frame

The RTK Station GNSS data will be contributed to IGS so as to realise a precise reference frame.

d) Transportation Navigation and Recreation

With the ultimate goal of providing data in real time, transportation monitoring is of great potential by providing real time positioning, vehicle tracking and reporting and setting up of the intelligent transport system. Fishing, boating, bike touring and hiking could be more enjoying just by knowing that you are constantly on course.

e) Aerial Photography Surveys

RTK data can be used by supplying post-process GNSS data in RINEX format. As such, aircraft conducting aerial photography work will be able to fly at any specified time without having to initiate the ground reference station.

2.3 ITRF2005 REALIZATION IN BRUNEI

The RTK network is a homogeneous covers the whole of Brunei Darussalam. The data from eight (8) RTK stations, between 17th May 2009 and 2nd June 2009 have been processed along with those from more than fifty IGS stations as part of the realization of the Zero Order Geodetic Network for Brunei Darussalam. The long-term objective is to integrate the network into the International Terrestrial Reference Frame (ITRF) based on the International Terrestrial Reference System (ITRS).

ITRF is realized through a set of station coordinates of global terrestrial fiducial points. The coordinates of the points are published by the International Earth Rotation Service (IERS) in its annual reports. The latest realization of ITRF series is the ITRF2005, released in 2008. This is the latest realization of the ITRF and thus the most accurate to date. It has the reference epoch on 1st January 2000 at 12:00 hrs.

2.3.1 Network Design

The Main Network comprises of the existing new GNSS Permanent stations in Brunei and Malaysia Permanent GNSS Network (MyRTKnet) and connected to the global reference system of International Terrestrial Reference Frame (ITRF) through selected International GNSS Services (IGS) stations. Further densification is to connect the main GNSS network and the existing Borneo Triangulation 1948 (BT48) points.

The Main Network with baseline length up to 3000 km will certainly need more observation data to realize the GDBD on the global reference frame. GNSS data for 17 days received from Survey Department were used to connect the Main Network to the ITRF2005 reference frame. However, the densification network with baseline vectors varies between 5 to 60 km only need a minimum of 24 hours of GNSS data in order to have high precision results. Separating the two networks will greatly reduce the observation times and number of GNSS instrument occupation without decreasing the anticipated accuracy.

The final combined adjustment have been carried out by stacking all daily normal equation from both networks and will be simultaneously realized on ITRF2005 reference frame.

2.3.1.1 Zero Order Network

a) IGS Stations

IGS, formerly the International GPS Service, is a voluntary federation of more than 200 worldwide agencies that pool resources and permanent GPS & GLONASS station data to generate precise GPS & GLONASS products. The IGS is committed to providing the highest quality data and products as the standard for Global Navigation Satellite Systems (GNSS) in support of Earth science research, multidisciplinary applications, and education. Currently the IGS includes two GNSS, GPS and the Russian GLONASS, and intends to incorporate future GNSS. You can think of the IGS as the highest-precision international civilian GPS community.

The International GPS Service is committed to providing the highest quality data and products as the standard for global navigation satellite systems (GNSS) in support of Earth science research, multidisciplinary applications, and education. These activities aim to advance scientific understanding of the Earth system components and their interactions, as well as to facilitate other applications benefiting society.

All IGS stations are equipped with strictly required equipment and operational characteristics. The GPS equipment, and its surroundings, must not be disturbed or changed unless a clear benefit outweighs the potential for discontinuities in the time series. Examples include, equipment failure, planned replacement of obsolete equipment and vendor-recommended firmware updates.

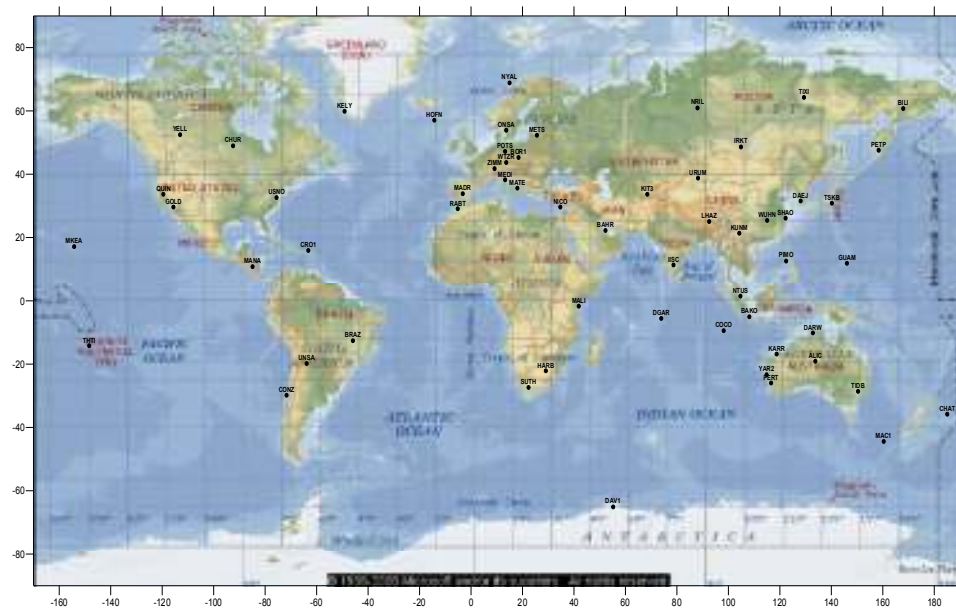


Figure 2.2: Selected IGS Stations

b) Brunei Darussalam RTK Stations

Eight (8) Brunei's existing GNSS permanent stations (Figure 2.1) also are part of the Main Network. These stations will be the backbone for the future active and passive GNSS network in Brunei.

c) Malaysian MyRTKnet Stations

Malaysia RTK Network (MyRTKnet) is a network of permanently running GPS base stations, at spacing from 30 to 100 km, feeding GPS data to a processing centre via a computer network. A central facility was setup to model the spatial errors that limit GPS accuracy through a network solution and generate corrections for roving receivers to be positioned anywhere inside the network with an accuracy better than a few centimetres to a few decimetres in real time. At the same time, a web site was made available to download GPS data for post-processing solutions.

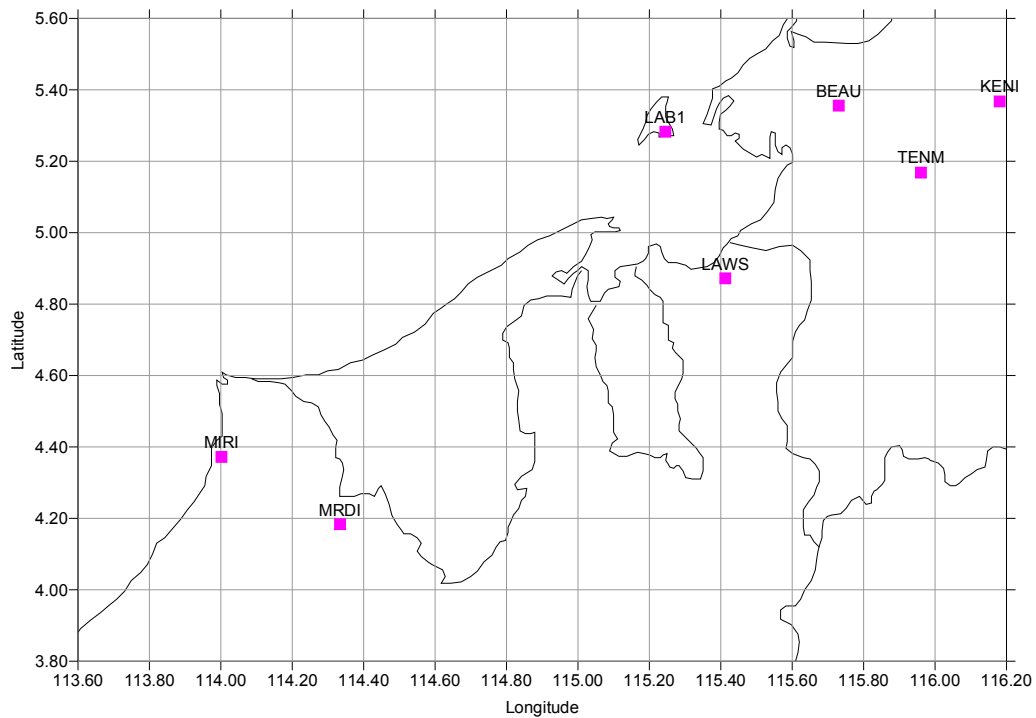


Figure 2.3: Selected MyRTKnet Station

2.3.1.2 Primary Geodetic Network

The Primary Geodetic Network comprised of the fifteen (15) stations from Zero Order Network (8 RTK and 7 MyRTKnet stations), thirteen (13) Primary Triangulation stations, and five (5) Secondary Triangulation station in Borneo Triangulation 1948 (BT48) system. The Primary Geodetic Network will be used to compute 7-Parameter transformation between ITRF frames to BT48 local system. The proposed BT48 triangulation stations distribution is as in Figure 2.4.

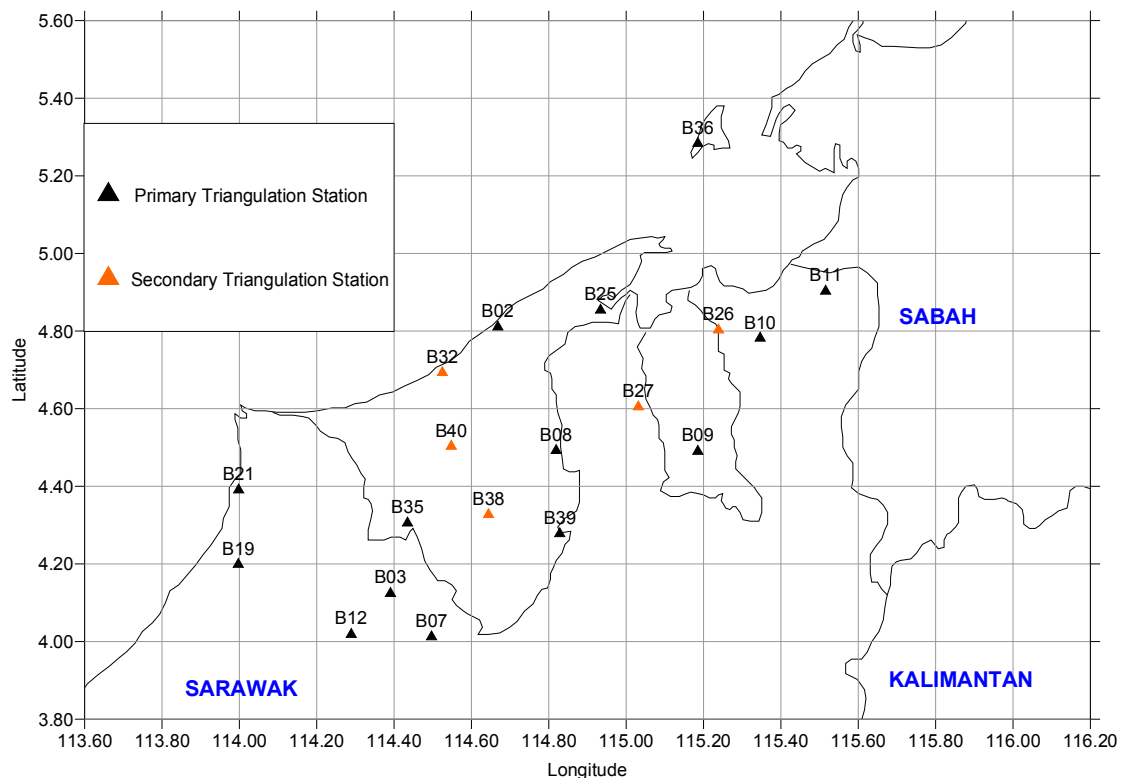


Figure 2.4: Primary Geodetic Network

2.3.2 Zero Order Network Data Processing

The Bernese GPS Software is a sophisticated tool meeting highest quality standards for geodetic and further applications using Global Navigation Satellite Systems (GNSS). Both of the currently active GNSS are supported: the American Global Positioning System (GPS) and the Russian Global Navigation Satellite System (GLONASS). Bernese is scientific software written by a team of geodetic scientists at the Astronomical Institute University of Berne (AUIB) in Switzerland. The software is the latest offering and a revised version of its predecessor developed by the geodetic team at the University of Berne. It combines specialized surveying knowledge with advanced software techniques. Data input and output is achieved via keyboard interface, allowing one to select commands and performs tasks with relative ease. The main features of the software are as follows:

- All GPS observables (code and phase) on L1 and L2 carrier frequencies and their different linear combinations may be used;
- All mathematical correlations and the combinations among the observables may be modeled;
- Baseline, session and network processing can be performed.

The Bernese software consists of a collection of batch and interactive programs that are executed for the following operations:

Transfer	to decode GPS observational data and satellite navigation data in RINEX into the Bernese format
Orbit	to create satellite orbits in Bernese format
Pre-Processing	to perform single point station positioning and correct cycle slips
Processing	to estimate parameters in baseline or network modes

Automation in the routine GNSS processing in order to obtain the RTK station coordinates was achieved using the Bernese Processing Engine (BPE) that runs on Microsoft Windows XP platform. Following are the main highlights of the GPS processing:

2.3.2.1 Pre-processing

The general strategy used for daily pre-processing is as follows:

- Use of IGS Final orbit referred to IGS05 and IGS final earth rotation parameter (ERP) series.
- Extrapolate IGS stations coordinates using ITRF2005 coordinates and velocity.
- Absolute Antenna phase center offset - Phas_COD.I05 table.
- Rinex to Bernese Conversion : 30 second data sampling.
- Conversion: IGS SP3 ephemeris - tabular format - Bernese format. Ocean tides correction - OT-SCRC model is introduced with development planetary ephemeris (DE200).
- Single Point Positioning using the L3 code pseudo-range measurement - estimate the receiver clock correction.
- Satellite clock biases - eliminated by forming double-difference observations.
- Forming single difference observation by using "shortest" method.
- Phase check using triple difference for data screening - to fix the cycle-slips and to mark the short data interval, gaps, unpaired observations and ambiguity setup.

2.3.2.2 Daily Solution

The computation of baseline and daily solutions is as follows:

- Adjustment of double difference carrier phase and ionosphere effect eliminated by the L3 linear combination.
- Zenith delay parameters were estimated once per 1-hours interval with Neill Mapping Function.
- Ambiguity resolution strategy: QIF (Quasi Ionosphere Free) [Mervart, 1995] ambiguity fixing strategy in baseline-by-baseline mode with global ionosphere model.
- About 90 % of the ambiguities were fixed using the above strategy.
- The daily solutions (session) of the independent baselines were loosely constrained with 1 m apriori sigma and no coordinates in the network adjustment were fixed.

2.3.2.3 Campaign Solution

The campaign solution was carried out as follows:

- Campaign solution: Combination of seventeen (17) normal equations of the daily solution.
- 53 IGS stations were held fixed with the coordinates transformed to an epoch of the middle of the campaign.
- Free Network adjustment with the introduction of three parameter Helmert Transformation had been applied for the campaign solution.
- Results were analyzed statistically for coordinate repeatability and RMS of residuals.
- Bad solutions were excluded at this stage.
- The RMS of campaign solution is less than 10 mm in the horizontal and height components.

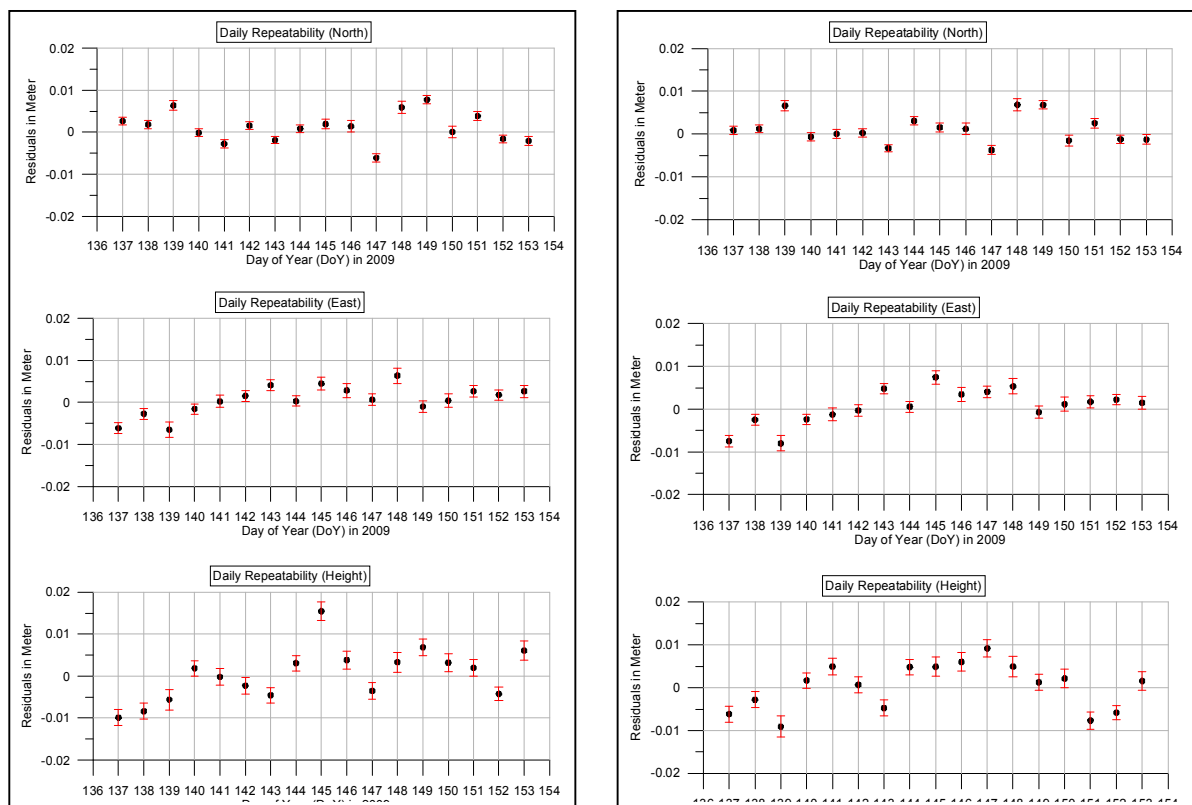
2.3.2.4 Results and Analyses

The final combined solution consists of 17 daily solutions with 64 stations (8 RTK stations, 49 IGS stations and 7 MyRTKnet stations). Minimally constraints adjustment using free network solution with 3-parameter helmert transformation was used to adjust the daily normal equation freely and transform them using forty one (41) selected IGS

station. This process will allow for the internal reliability investigation and to detect outliers. With short data span, the introduction of reference velocity for the fixed stations is not possible; hence, the final coordinates for all stations were fixed at the middle of the observation epoch.

Figure 2.5 - Figure 2.12 show the daily repeatability of 3-dimensional stations coordinates for RTK stations. RMS of residuals is between 2.78 - 3.73 mm, 3.50 - 6.27 mm and 4.89 - 10.15 mm for northing, easting and height components respectively. It can be concluded that the internal accuracy of the Brunei Darussalam RTK stations from the free network adjustment is 2 to 7 mm for the horizontal component and 4 to 10 mm in the height.

Comparison of IGS stations coordinates has been made in order to determine the accuracy of the network with respect to the ITRF2005 reference frame. The final combined coordinate from the network adjustment was fixed at 25 May 2009 (\approx 1st June 2009) or 2009.45, the reference coordinates (ITRF2005 Epoch 2000.0) for the IGS stations were propagated on the same epoch as the adjusted coordinates. The RMS of fitting is 5.6 mm, 5.1 mm and 6.3 mm for the northing, easting and height components. It can be concluded that the accuracy of Brunei Darussalam RTK stations with respect to the ITRF2005 reference frame with free network strategy is 7 to 12 mm in the horizontal component and 11 to 16 mm in height.



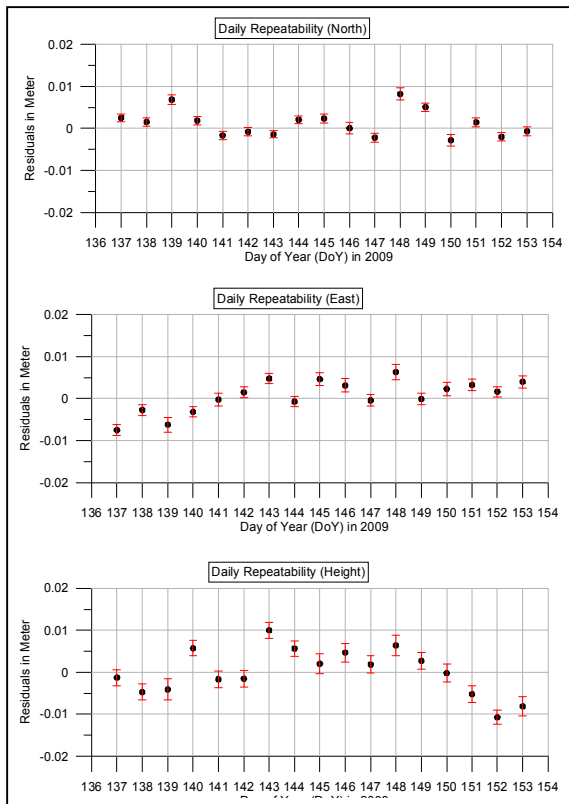


Figure 2.7: Daily Repeatability (TEMB)

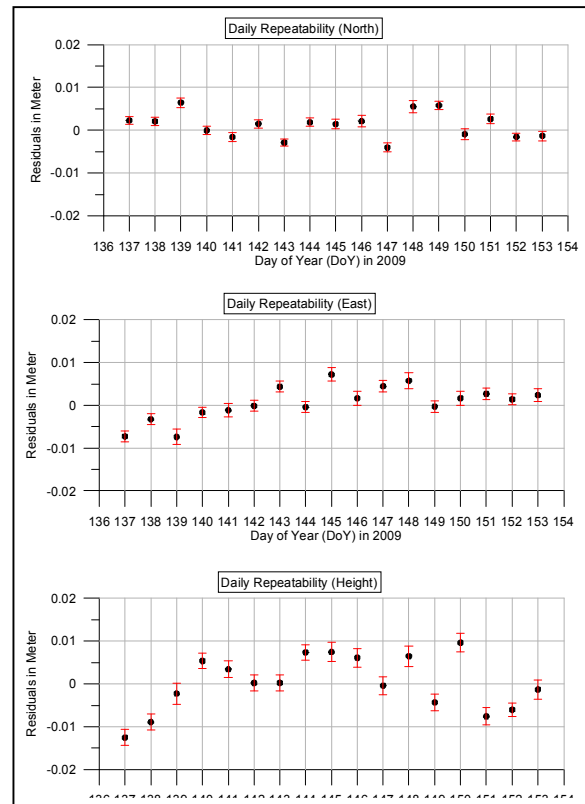


Figure 2.8: Daily Repeatability (MURA)

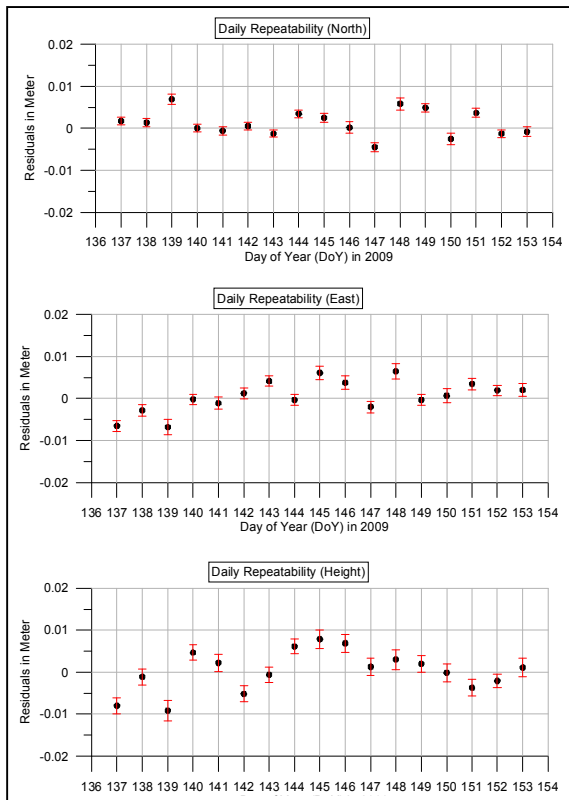


Figure 2.9: Daily Repeatability (LIAN)

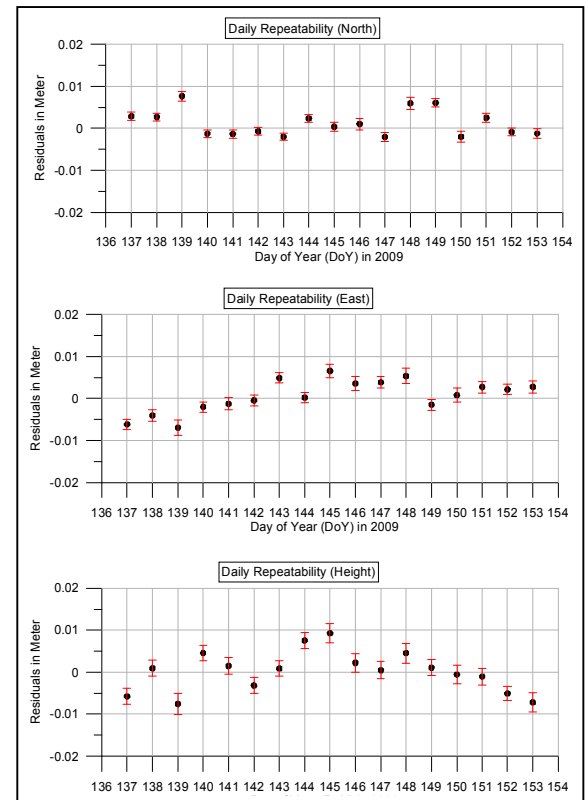


Figure 2.10: Daily Repeatability (LAMU)

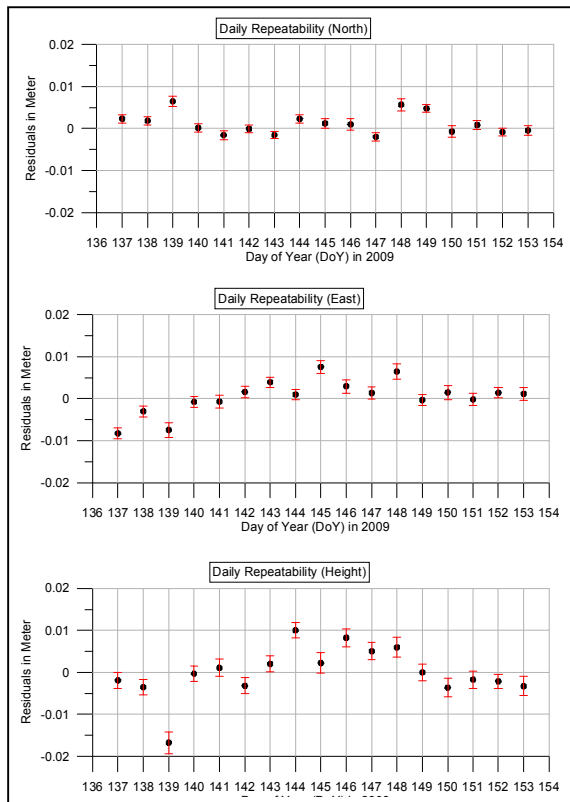


Figure 2.11: Daily Repeatability (KBEL)

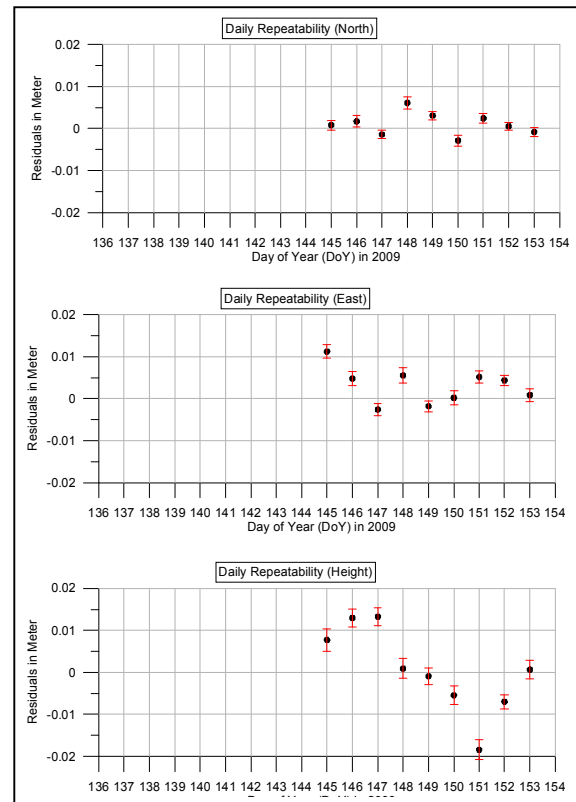


Figure 2.12: Daily Repeatability (LABI)

2.3.3 Primary Geodetic Network Data Processing

Establishment of Primary Geodetic Network is aimed to derive transformation parameter between GDBD2009 and the existing system of BT48. The GNSS observation of the network has followed the guidelines and procedures as Zero Order Network. Observation on each triangulation station was carried out for 48 hours with calibrated GNSS equipment and accessories. GNSS observation was scheduled in February and March 2009.

The campaign solution was carried out as follows:

- Campaign solution: Combination of ten (10) normal equations of the daily solution.
- 15 Zero Order Network stations were held fixed with the coordinates referred to 1st June 2009.
- Free Network adjustment with the introduction of three parameter Helmert Transformation had been applied for the campaign solution.

- Results were analyzed statistically for coordinate repeatability and RMS of residuals.
- The RMS of campaign solution is less than 10 mm in the horizontal and height components.

2.3.3.1 Results and Analyses

The final combined solution consists of 10 daily solutions with 28 stations (10 Zero Order Network stations, and 18 Primary Geodetic Network stations). Minimally constraints adjustment using free network solution with 3-parameter helmert transformation was used to adjust the daily normal equation freely and transform them using ten (10) Zero Order Network station.

RMS of residuals is between 0.5 - 4 mm and 2 – 10 mm for the horizontal and vertical components respectively. It can be concluded that the accuracy of the Primary Geodetic Network stations from the free network adjustment is within 10 mm.

CHAPTER 3

TRANSFORMATION OF COORDINATES

3.1 INTRODUCTION

Today, the modern geodetic datum range from flat Earth models used for plane surveying to very complex models used for global applications which completely describe the size, shape, orientation, gravity field, angular velocity of the Earth and others. With satellite positioning that is especially GNSS with rapidly increasing applications, the relationship between various geodetic datum become very necessary and important. With the widespread use of GPS, there is a trend for working datum to be consistent with the ITRF and WGS84. Therefore, accurate positioning using satellite-based systems need the full understanding of reference frame conversions or transformations.

Having data in one datum and needing the coordinates in other is a common occurrence. It happens routinely when you use a GNSS receiver for positioning purposes. It may be hidden from the user, but a transformation must occur to display coordinates from a GNSS receiver in any other datum than WGS84. Of course, map and chart makers need to often make these transformations. Re-surveying everything each time a datum change is needed is impractical. Over any small area, say 100 km or so, the transformation will be just a constant shift in latitude, longitude and height. This is a practical statement, and as accuracy requirements increase, the area over which a simple offset can be used gets smaller.

There are common methods of making these transformations from one datum to another. In the science world, the transformation is often viewed from a vector perspective. The coordinates are transformed from Cartesian Earth Centered Earth

Fixed (ECEF) XYZ values of one datum to another. If latitude, longitude and height (PLH) are given or needed, the conversion to ECEF is done before the vector mathematics and then the new coordinates are converted back to PLH.

Therefore, the complete datum conversion is usually based on seven parameter transformations, which include three translation parameters, three rotation parameters and a scale. Veis (1960), Bursa-Wolf (1963), Molodensky (1962), Vanicek and Wells (1974) and many others have developed the transformation of 3-D co-ordinate system, for transforming geodetic datum. This method is usually called a Helmert transformation.

3.2 MATHEMATICAL MODELS

3.2.1 Curvilinear To Cartesian Coordinates Conversion

Three-dimensional co-ordinates could be converted from Cartesian to curvilinear or vice versa through the knowledge of the parameters of an adopted reference ellipsoid. The forward transformation from geodetic co-ordinates (ϕ, λ, h) to Cartesian co-ordinates (X, Y, Z) is given in Heiskanen and Moritz (1967), p182 as:

$$X = (N + h) \cos \phi \cos \lambda \quad (1)$$

$$Y = (N + h) \cos \phi \sin \lambda \quad (2)$$

$$Z = (N(1 - e^2) + h) \sin \phi \quad (3)$$

where, the prime vertical radius of curvature (N) is:

$$N = \frac{a}{\sqrt{1 - e^2 \sin^2 \phi}} \quad (4)$$

with:

- a : the semi-major axis of the reference ellipsoid;
- e : the first eccentricity of the reference ellipsoid.

The reverse transformation from Cartesian to curvilinear co-ordinates is more involved because the direct inversion of equations (1), (2) and (3) requires some iteration for the solution of latitude (Heiskanen and Moritz (1967)). The iterative reverse transformation

from Cartesian co-ordinates (X, Y, Z) to geodetic co-ordinates (ϕ, λ, h) is given in Heiskanen and Moritz (1967) as:

$$\phi = \arctan \left[\frac{Z}{p} \left(1 - e^2 \cdot \frac{N}{N+h} \right)^{-1} \right] \quad (5)$$

$$\lambda = \arctan \left[\frac{Y}{X} \right] \quad (6)$$

$$h = \frac{p}{\cos \phi} - N \quad (7)$$

With initial value base on;

$$\tan \phi_0 = \left(\frac{Z}{p} \right) \left[\frac{1}{1 - e^2} \right] \quad (8)$$

$$N_0 = \frac{a}{(1 - e^2 \cdot \sin^2 \phi_0)^{\frac{1}{2}}} \quad (9)$$

$$p = \sqrt{X^2 + Y^2} \quad (10)$$

3.2.2 Bursa-Wolf Model

The Bursa-Wolf is a seven-parameter model for transforming three-dimensional Cartesian co-ordinates between two datums. This transformation model is more suitable for satellite datums on a global scale (Krawisky and Thomson, 1974). The transformation involves three geocentric datum shift parameters $(\Delta X, \Delta Y, \Delta Z)$, three rotation elements (R_X, R_Y, R_Z) and scale factor $(1 + \Delta L)$.

$$\begin{bmatrix} X_{To} \\ Y_{To} \\ Z_{To} \end{bmatrix} = \begin{bmatrix} \Delta X \\ \Delta Y \\ \Delta Z \end{bmatrix} + \begin{bmatrix} 1 + \Delta L & R_Z & -R_Y \\ -R_Z & 1 + \Delta L & R_X \\ R_Y & -R_X & 1 + \Delta L \end{bmatrix} \begin{bmatrix} X_{From} \\ Y_{From} \\ Z_{From} \end{bmatrix} \quad (11)$$

where;

X_{To}, Y_{To}, Z_{To} : are the global datum Cartesian coordinates;

$X_{From}, Y_{From}, Z_{From}$: are the local datum Cartesian co-ordinates.

Transformation formula (11) is only suitable for small rotation angle between the datums. The inverse transformation can be carried out by simply inverting the parameters (translations, rotation and scale). Datum transformation with the value of rotation angle is significant, strict inverse formula need to be used.

Bursa-Wolf mathematical model that has been used globally is:-

$$\begin{pmatrix} X \\ Y \\ Z \end{pmatrix}_{To} = \begin{pmatrix} \Delta X \\ \Delta Y \\ \Delta Z \end{pmatrix} + (1 + \delta) \mathbf{R} \begin{pmatrix} X \\ Y \\ Z \end{pmatrix}_{From} \quad (12)$$

where R is the rotation matrix define by:-

$$\mathbf{R} = \mathbf{R}_z \mathbf{R}_y \mathbf{R}_x = \begin{pmatrix} \cos \omega_z & \sin \omega_z & 0 \\ -\sin \omega_z & \cos \omega_z & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \cos \omega_y & 0 & -\sin \omega_y \\ 0 & 1 & 0 \\ \sin \omega_y & 0 & \cos \omega_y \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos \omega_x & \sin \omega_x \\ 0 & -\sin \omega_x & \cos \omega_x \end{pmatrix} \quad (13)$$

where R (3 x 3),

$$\begin{aligned} R_{11} &= \cos(Ry) \cdot \cos(Rz) \\ R_{12} &= \cos(Rx) \cdot \sin(Rz) + \sin(Rx) \cdot \sin(Ry) \cdot \cos(Rz) \\ R_{13} &= \sin(Rx) \cdot \sin(Rz) - \cos(Rx) \cdot \sin(Ry) \cdot \cos(Rz) \\ R_{21} &= -\cos(Ry) \cdot \sin(Rz) \\ R_{22} &= \cos(Rx) \cdot \cos(Rz) - \sin(Rx) \cdot \sin(Ry) \cdot \sin(Rz) \\ R_{23} &= \sin(Rx) \cdot \cos(Rz) + \cos(Rx) \cdot \sin(Ry) \cdot \sin(Rz) \\ R_{31} &= \sin(Ry) \\ R_{32} &= -\sin(Rx) \cdot \cos(Ry) \\ R_{33} &= \cos(Rx) \cdot \cos(Ry) \end{aligned}$$

The strict inverse formula for reverse transformation is as follows:-

$$\begin{pmatrix} X \\ Y \\ Z \end{pmatrix}_{To} = \frac{\mathbf{R}^{-1}}{(1 + \delta)} \left[\begin{pmatrix} X \\ Y \\ Z \end{pmatrix}_{From} - \begin{pmatrix} \Delta X \\ \Delta Y \\ \Delta Z \end{pmatrix} \right] \quad (14)$$

With R^{-1} is the inverse of rotation matrix (orthogonal matrix), which is similar to the transpose of rotation matrix (R^T). Most of the commercial transformation software does support the strict inverse formula, if not, another set of transformation parameter need to be derived.

In order to convert the geocentric coordinates of XYZ to the geodetic coordinate of PLH, ellipsoid properties for the respective datums are listed below:

Table 3.1: Ellipsoid Properties

No.	Ellipsoid	a (m)	1/f (m)	Ref. Frame
1	GRS80	6378137.000	298.2572221	ITRF91 - 2000
2	WGS84	6378137.000	298.2572236	WGS84
3	Mod. Everest (Brunei)	6377298.556	300.8017	BT48

3.2.3 Molodensky-Badekas

The Molodensky-Badekas (Molodensky et al., 1962; Badekas, 1969) is also a seven parameter model or sometimes called 10 parameter models (when taking into account the centroid coordinates) for transforming three-dimensional Cartesian co-ordinates between two datums. This transformation model is more suitable for transformation between terrestrial and satellite datums, (Krakwisky and Thomson, 1974). The transformation also involves three barycentric datum shift parameters (dX , dY , dZ), three rotation elements (R_X , R_Y , R_Z) and scale factor ($1 + \Delta L$). This transformation model is theoretically identical to the Bursa-Wolf model. The model in its matrix-vector form could be written as (see Burford 1985):

$$\begin{bmatrix} X_{To} \\ Y_{To} \\ Z_{To} \end{bmatrix} = \begin{bmatrix} dX \\ dY \\ dZ \end{bmatrix} + \begin{bmatrix} X_m \\ Y_m \\ Z_m \end{bmatrix} + \begin{bmatrix} 1 + \Delta L & R_Z & -R_Y \\ -R_Z & 1 + \Delta L & R_X \\ R_Y & -R_X & 1 + \Delta L \end{bmatrix} \begin{bmatrix} X_{From} - X_m \\ Y_{From} - Y_m \\ Z_{From} - Z_m \end{bmatrix} \quad (15)$$

where;

X_{To}, Y_{To}, Z_{To} : are the global datum Cartesian coordinates;

$X_{From}, Y_{From}, Z_{From}$: are the local datum Cartesian coordinates.

X_m, Y_m, Z_m : are the centroid coordinates.

3.2.4 GDBD2009 TO BT48 PARAMETER DERIVATION

Bursa-Wolf transformation model has been adopted to transform GDBD2009 and BT48 coordinates with a single set of seven parameter transformation. The use of Molodensky-Badekas transformation model is not considered due to the model is not reversible with single set of 10 transformation parameter. This is because of the centroid is derive from the source reference system and the rotations are derived about this point. In order not to complicate the process especially when it involve cadastral survey, the use of Bursa-Wolf seven parameter models is more suitable for Brunei Darussalam.

3.2.4.1 Derivation Flow

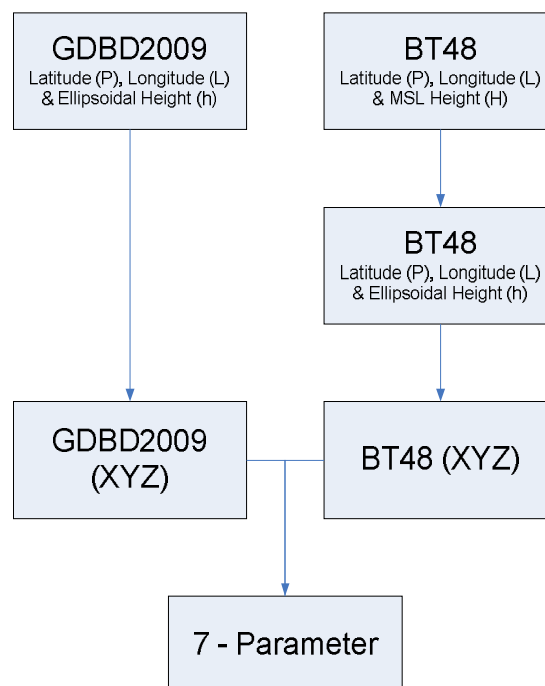


Figure 3.1: Parameter Derivation Flow

3.2.4.2 Common Points

Eighteen (18) Borneo Triangulation 1948 (BT48) points used to derive the transformation parameter are listed as in Table 3.2 and Table 3.3. Mean Sea Level (MSL) height for all BT48 triangulation points are not available for certain stations. In addition, the MSL height value is not in the original triangulation list. With common

points distribution covered Brunei Darussalam, Sarawak and Sabah in Malaysia, the MSL value probably has been updated by the local authority using local vertical reference system.

In order to have homogeneous MSL height value for BT48 reference system, Earth Geopotential Model 2008 (EGM2008) has been used to derive the MSL value using GDBD2009 coordinates.

Table 3.2: GDBD2009 Coordinates for Common Points

No	Station Name	Latitude			Longitude			Ellipsoidal Height (m)
		°	'	"	°	'	"	
B001	Agok	4	54	10.81608	114	47	40.48858	159.552
B004	Marudi West Base	4	08	12.51469	114	20	17.37041	61.436
B007	Batu Belah	4	00	43.97811	114	29	48.80825	336.963
B008	Bedawan	4	29	31.93600	114	49	08.16156	574.667
B010	Berayong	4	46	54.19496	115	20	46.72692	772.482
B011	Bumbung Rumah	4	54	10.04586	115	30	56.29515	931.992
B012	Kala	4	01	06.10192	114	17	21.67965	138.429
B019	Lambir	4	11	56.15276	113	59	50.05005	506.891
B021	Miri	4	23	26.17784	113	59	53.25767	124.704
B025	Saeh	4	51	14.25856	114	56	01.30362	263.166
B026	Sagan A	4	48	10.09279	115	14	19.01036	103.106
B027	Sagan B	4	36	16.75911	115	01	53.09025	580.949
B032	Tunggulian	4	41	34.62829	114	31	29.42203	125.499
B033	Telingan	4	22	28.38573	114	28	16.39018	300.629
B034	Tempayan Pisang	5	00	36.27763	115	02	57.29423	190.806
B036	Bukit Timbalai	5	17	00.40526	115	11	07.16295	114.739
B038	Tunjang Pipit	4	19	37.14257	114	38	39.05205	96.369
B039	Ulu Tutong	4	16	40.52899	114	49	41.99909	422.082

Table 3.3: BT48 Coordinates for Common Points

No	Station Name	Latitude			Longitude			Orthometric Height (EGM2008) (m)
		°	'	"	°	'	"	
B001	Agok	4	54	13.83900	114	47	29.55300	116.349
B004	Marudi West Base	4	08	15.34450	114	20	6.20820	18.087
B007	Batu Belah	4	00	46.76820	114	29	37.72420	292.629
B008	Bedawan	4	29	34.84710	114	48	57.23800	530.537
B010	Berayong	4	46	57.19470	115	20	36.06160	726.141
B011	Bumbung Rumah	4	54	13.08680	115	30	45.71410	885.300
B012	Kala	4	01	8.89890	114	17	10.49540	94.805
B019	Lambir	4	11	59.00070	113	59	38.72350	465.218
B021	Miri	4	23	29.06850	113	59	41.92890	83.991
B025	Saeh	4	51	17.26840	114	55	50.43650	219.263
B026	Sagan A	4	48	13.10000	115	14	8.29000	57.417
B027	Sagan B	4	36	19.70500	115	01	42.26900	535.620
B032	Tunggulian	4	41	37.58700	114	31	18.35000	83.316
B033	Telingan	4	22	31.27400	114	28	5.30300	257.629
B034	Tempayan Pisang	5	00	39.33280	115	02	46.47970	146.863
B036	Bukit Timbalai	5	17	3.54830	115	10	56.40880	71.144
B038	Tunjang Pipit	4	19	40.01550	114	38	28.04610	52.563
B039	Ulu Tutong	4	16	43.46170	114	49	30.97870	377.189

3-Dimensional datum transformation use ellipsoidal height instead of MSL height value in the geodetic datum transformation computation. For the ellipsoidal solution, the MSL heights (H) have to be converted into their corresponding ellipsoidal heights (h) on the Modified Everest Spheroid. The basic equation which relates the ellipsoidal height (h), the orthometric height (H) and the geoid height (N) is given by:

$$h = H + N \quad (16)$$

To compute the N value on Modified Everest ellipsoid, "Multiple Regression Equation for DMA-Developed Local Geoid Height on Timbalai 1948 Datum" has been used.

$$N_{BT48} = -1.703 - 6.806.U - 7.143.V + 18.663.U^3 + 23.300.UV^3 - 13.211.U^5 - 10.642.U^4V - 1.909.V^5 - 36.586.U^3V^5 - 28.381.U^3V^7 \quad (17)$$

where,

$$\begin{aligned} U &= K (\phi - 4) \\ V &= K (\lambda - 114) \\ K &= 0.20943951 \end{aligned}$$

The final BT48 coordinates are listed in Table 6.5.

Table 3.4: BT48 Coordinates for Common Points

No	Station Name	Latitude			Longitude			Ellipsoidal Height (m)
		°	'	"	°	'	"	
B001	Agok	4	54	13.83900	114	47	29.55300	112.314
B004	Marudi West Base	4	08	15.34450	114	20	6.20820	015.687
B007	Batu Belah	4	00	46.76820	114	29	37.72420	290.169
B008	Bedawan	4	29	34.84710	114	48	57.23800	526.943
B010	Berayong	4	46	57.19470	115	20	36.06160	721.473
B011	Bumbung Rumah	4	54	13.08680	115	30	45.71410	880.298
B012	Kala	4	01	8.89890	114	17	10.49540	092.647
B019	Lambir	4	11	59.00070	113	59	38.72350	463.241
B021	Miri	4	23	29.06850	113	59	41.92890	081.748
B025	Saeh	4	51	17.26840	114	55	50.43650	215.082
B026	Sagan A	4	48	13.10000	115	14	8.29000	052.870
B027	Sagan B	4	36	19.70500	115	01	42.26900	531.581
B032	Tunggulian	4	41	37.58700	114	31	18.35000	079.904
B033	Telingan	4	22	31.27400	114	28	5.30300	254.701
B034	Tempayan Pisang	5	00	39.33280	115	02	46.47970	142.371
B036	Bukit Timbalai	5	17	3.54830	115	10	56.40880	066.265
B038	Tunjang Pipit	4	19	40.01550	114	38	28.04610	049.443
B039	Ulu Tutong	4	16	43.46170	114	49	30.97870	373.864

3.2.4.3 Analyses

Two (2) iterations have been carried out for transformation parameter derivation. Results and statistics from the first iteration are as follow:

Dx	=	681.26420	+ -	26.35870	m
Dy	=	-617.16822	+ -	16.20935	m
Dz	=	59.73994	+ -	32.98605	m
Rx	=	-0.31964	+ -	0.96960	"
Ry	=	1.26869	+ -	0.64711	"
Rz	=	-0.62341	+ -	0.88348	"
S	=	-7.29778	+ -	2.23036	ppm

Stand. Error of unit Weight So = 0.584

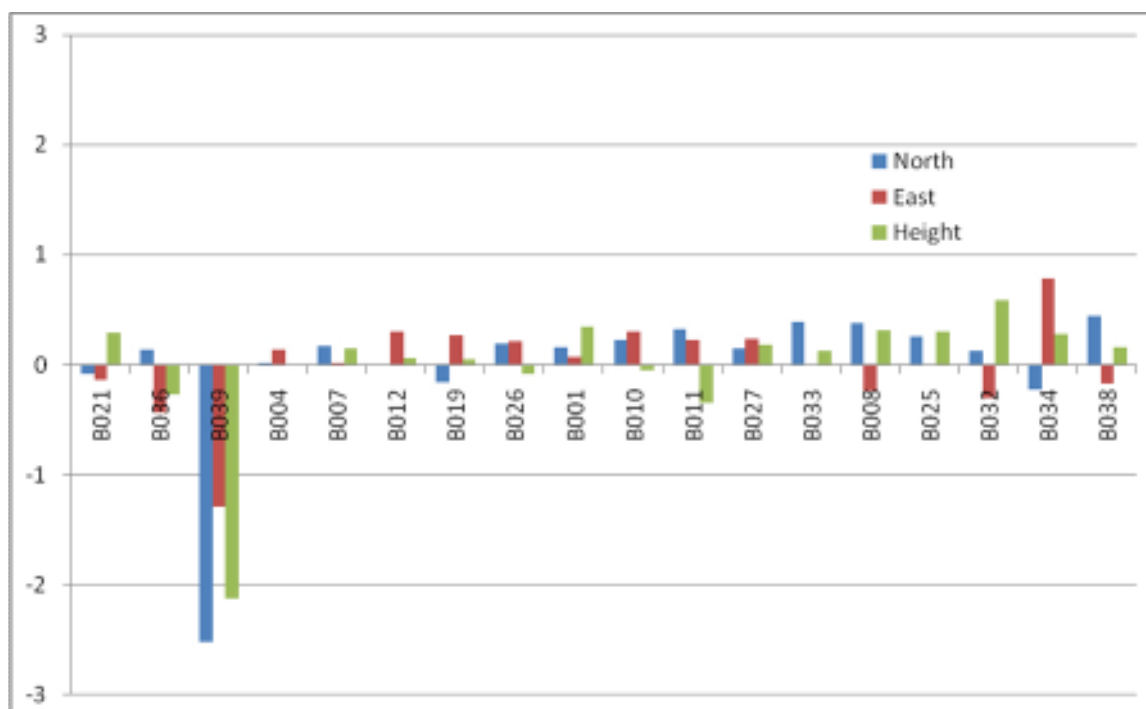


Figure 3.2: Common Points Residuals in meter (1st Iteration)

The results reveal that the scale factor between GDBD2009 and BT48 is at -7.297 ppm with the standard error of unit weight of 0.584. Even though the results look promising, however the residuals plot for common points has shown that BT48 stations B039 (Ulu Tutong) has large residuals in all components.

Second iteration has excluded B039, and the results are as follow:

Dx	=	689.59370	+ -	11.31003	m
Dy	=	-623.84046	+ -	6.78722	m
Dz	=	65.93566	+ -	14.40584	m
Rx	=	-0.02331	+ -	0.42256	"
Ry	=	1.17094	+ -	0.27705	"
Rz	=	-0.80054	+ -	0.37907	"
S	=	-5.88536	+ -	0.92598	ppm

Stand. Error of unit Weight So = 0.240

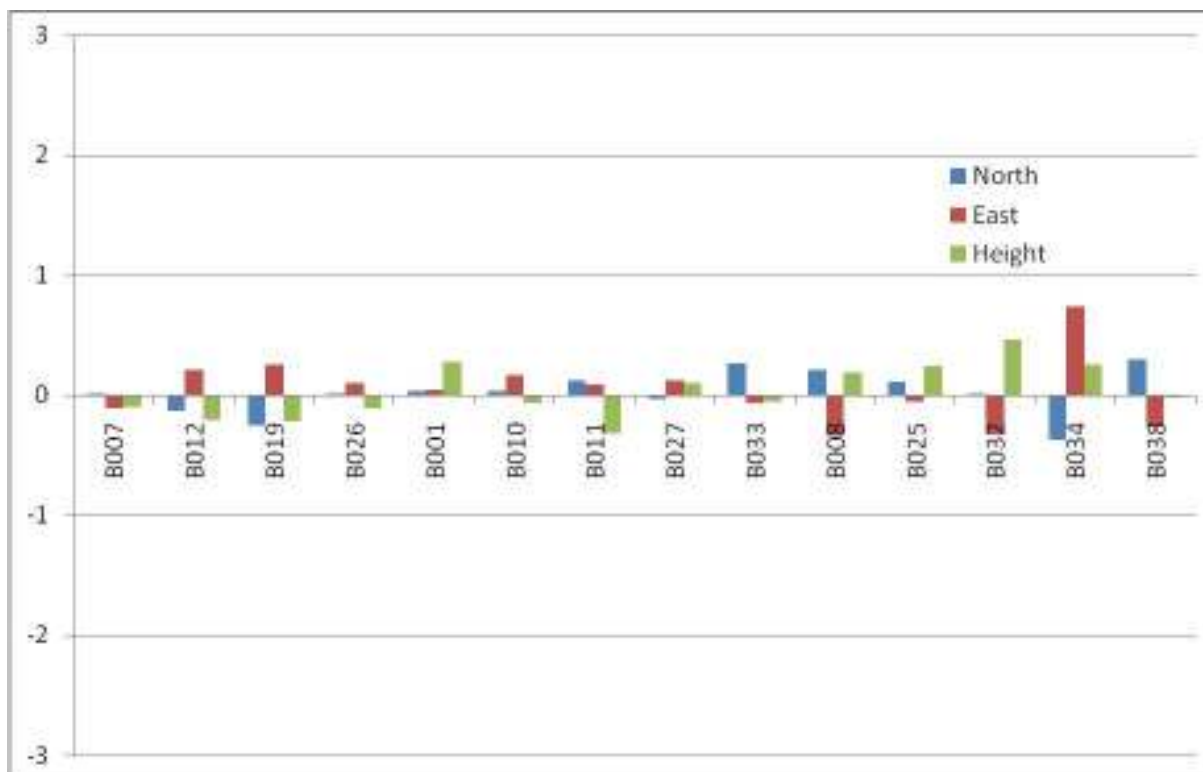


Figure 3.3: Common Points Residuals in meter (2nd Iteration)

Excluding B039 has improved the results drastically. Standard error of unit weigh is 0.240 residuals for common point are less than 0.5 meter. Results from second iteration are adopted as the final transformation parameter between GDBD2009 and BT48.

3.3 GDBD2009 TO WGS84 PARAMETER DERIVATION

Parameter derivation between GDBD2009 and WGS84 is using a simple three (3) parameter transformation model. It involves only the translation in X, Y and Z axis.

3.3.1 Derivation Flow

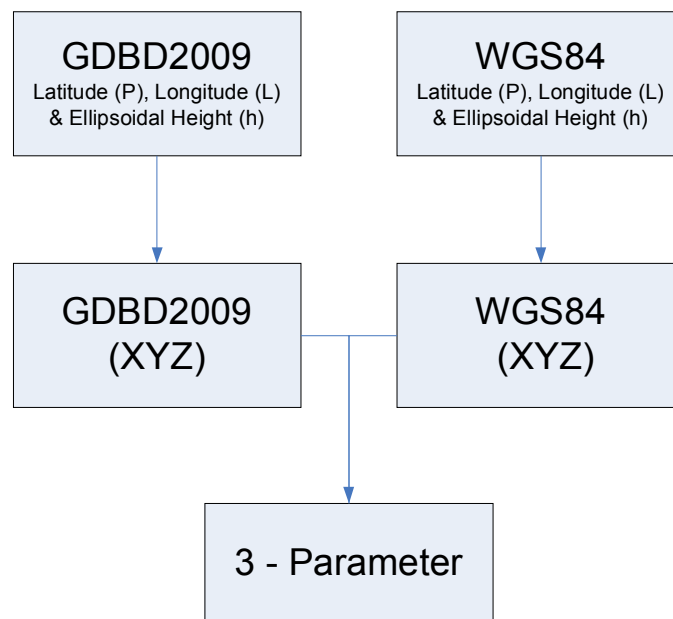


Figure 3.4: GDBD2009 to WGS84 Parameter Derivation Flow

3.3.2 Common Points

Three (3) World Geodetic System 1984 (WGS84) points was provided by Survey Department, Brunei Darussalam. The stations namely UKUR (Ukur), TEMB (Temburong) and TUTO (Tutong) are the existing GNSS permanent station in Brunei. The geodetic coordinates in WGS84 (PLh) was converted to their respective cartesian coordinates and listed in Table 3.5.

Table 3.5: GDBD2009 Coordinates for Common Points

STN	GDBD2009 Coordinates			WGS84 Coordinates		
UKUR	-2678449.0376	5762777.6676	543962.4543	-2678448.8387	5762777.7592	543962.5393
TUTO	-2651866.1282	5776287.9356	530901.1137	-2651865.9767	5776288.2923	530901.0653
TEMB	-2693842.9146	5757787.9060	520314.6162	-2693842.8596	5757787.8378	520314.6545

3.3.3 Analyses

A single iteration has been carried out to determine the transformation parameter. Results and statistics are as follow:

$D_x = 0.13513 \pm 0.07889 \text{ m}$
 $D_y = 0.12670 \pm 0.07889 \text{ m}$
 $D_z = 0.02497 \pm 0.07889 \text{ m}$

Stand. Error of unit Weight So = 0.137

Residuals

Point	N	E	U
TEMB	-0.002	0.155	-0.143
UKUR	-0.055	-0.043	-0.064
TUTO	0.056	-0.111	0.208

The results have shown that there are very good agreement between the two datum with standard error of unit weight is at 0.137. The residuals for the common point are less than 20 cm in all components.

3.4 MAP PROJECTION

The RSO is an oblique Mercator projection developed by Hotine in 1947 (Snyder, 1984). Hotine called the projection as "rectified skew orthomorphic". This projection is orthomorphic (conformal) and cylindrical. All meridians and parallel are complex curves. Scale is approximately true along a chosen central line (exactly true along a great circle in its spherical form). It is thus a suitable projection for an area like Switzerland, Italy, New Zealand, Madagascar, and Malaysia as well.

The RSO provide an optimum solution in the sense of minimizing distortion whilst remaining conformal for Brunei/Borneo. Table 3.6 tabulates the new *geocentric RSO* parameters for Brunei.

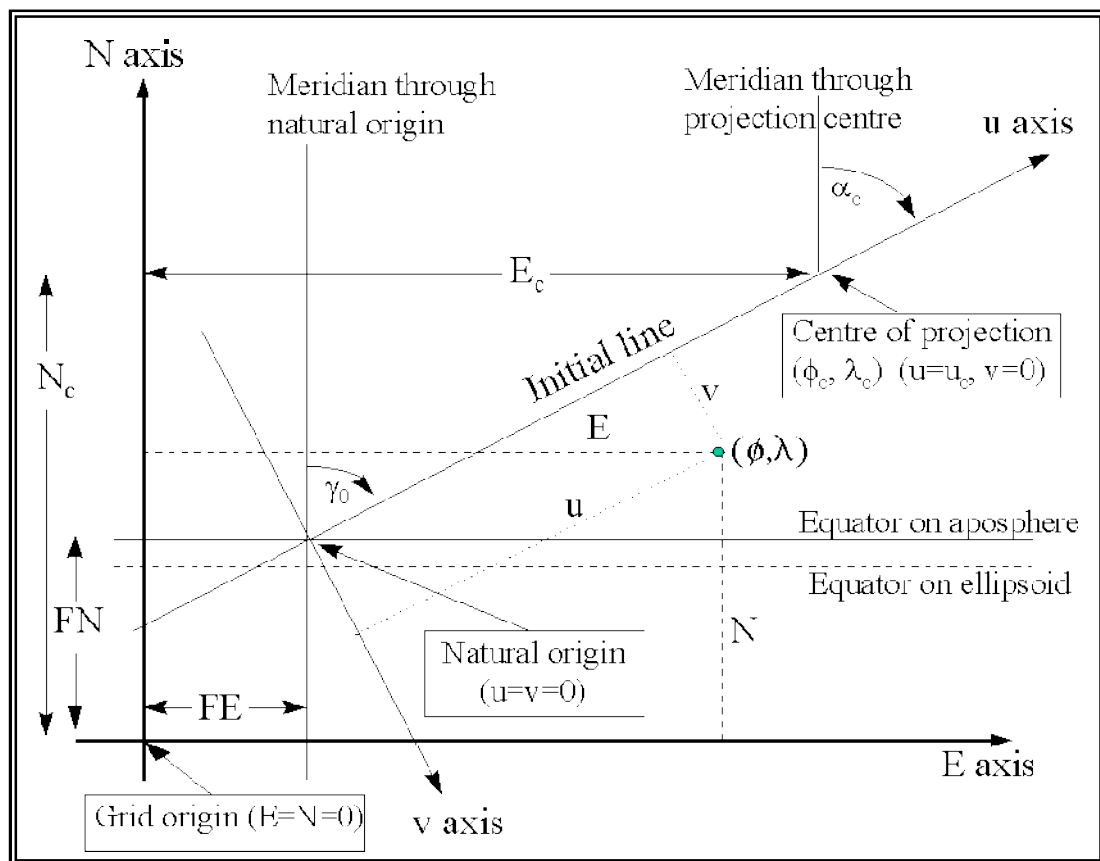


Figure 3.5: Hotine, 1947 (Snyder, 1984), Oblique Mercator (Source:EPSG)

Table 3.6: The New Geocentric RSO Projection Parameters

Parameter	Value
Ellipsoid	GRS 80
Semi-Major axis, a	6378137.000 Meters
Flattening, $1/f$	298.2572221
Latitude of Center of the Projection, ϕ_c	4° 00' 00" N
Longitude of Center of the Projection, λ_c	115° 00' 00" E
Rectified to Skew Grid, γ_o	$\sin^{-1}(0.8)$
Azimuth of Central Line, α_c	53° 18' 56.91582"
Scale factor, k_c	0.99984
False Origin (Easting)	Nil
False Origin (Northing)	Nil

The notation adopted for use in this section is as follows:

- ϕ_c = latitude of center of the projection.
 λ_c = longitude of center of the projection.
 α_c = azimuth (true) of the center line passing through the center of the projection.
 γ_c = rectified bearing of the center line.
 k_c = scale factor at the center of the projection.
 ϕ = geographical latitude
 λ = geographical longitude
 a = semi major axis of ellipsoid
 b = semi minor axis of ellipsoid
 f = flattening of ellipsoid

$$f = \frac{(a - b)}{a}$$
 e = eccentricity of ellipsoid

$$e^2 = \frac{(a^2 - b^2)}{a^2}$$

$$e_1^2 = \frac{(a^2 - b^2)}{b^2}$$
 N = Northing map coordinate

E	=	Easting map coordinate
FE	=	False Easting at the natural origin.
FN	=	False Northing at the natural origin.

Constants of the projection

$$B = \sqrt{\left[\frac{1 + e^2 \cdot \cos^4 \phi_c}{1 - e^2} \right]}$$

$$A = \frac{\sqrt{a \cdot B \cdot k_c (1 - e^2)}}{1 - e^2 \sin^2 \phi_c}$$

$$t_0 = \frac{\tan\left(\frac{\pi}{4} - \frac{\phi_c}{2}\right)}{\left[\frac{1 - e \sin \phi_c}{1 + e \sin \phi_c} \right]^{\frac{e}{2}}}$$

$$D = \frac{B \cdot \sqrt{1 - e^2}}{\cos \phi_c \cdot \sqrt{e^2 \sin^2 \phi_c}}$$

To avoid problems with computation of F, if $D < 1$, make $D^2 = 1$.

$$F = D + \text{SIGN}(\phi_c) \cdot \sqrt{D^2 - 1}$$

$$H = F \cdot t_0^B$$

$$G = \frac{F - \frac{1}{F}}{2}$$

$$\gamma_0 = \sin^{-1}(-0.6)$$

$$\lambda_0 = \lambda_c - \frac{\sin^{-1}(G \cdot \tan \gamma_0)}{B}$$

3.4.1 Conversion of Geographicals to Rectangulares and vice versa

3.4.1.1 Forward Case: To compute (E, N) from a given (ϕ , λ):

$$t = \frac{\tan\left(\frac{\pi}{4} - \frac{\phi}{2}\right)}{[(1 - e \sin \phi) / (1 + e \sin \phi)]^{e/2}}$$

$$Q = H / t^B$$

$$S = \frac{Q - 1/Q}{2}$$

$$T = \frac{Q + 1/Q}{2}$$

$$V = \sin[B(\lambda - \lambda_0)]$$

$$U = \frac{S \sin \gamma_0 - V \cos \gamma_0}{T}$$

$$v = \frac{A \ln\left(\frac{1-U}{1+U}\right)}{2 \cdot B}$$

For the Hotine Oblique Mercator (where the FE and FN values have been specified with respect to the origin of the (u, v) axes):

$$u = \frac{A}{B} \arctan\left(\frac{S \cos \gamma_0 + V \sin \gamma_0}{\cos[B(\lambda - \lambda_0)]}\right)$$

The rectified skew co-ordinates are then derived from:

$$E = v \cos \gamma_c + u \sin \gamma_c + (FE \text{ or } E_c)$$

$$N = u \cos \gamma_c - v \sin \gamma_c + (FN \text{ or } N_c)$$

3.4.1.2 Reverse case: Compute (ϕ , λ) from a given (E, N):

For the Hotine Oblique Mercator:

$$v' = (E - FE) \cos \gamma_c - (N - FN) \sin \gamma_c$$

$$u' = (N - FN) \cos \gamma_c + (E - FE) \sin \gamma_c$$

Then the other parameters can be calculated.

$$Q' = \exp[-(Bv' / A)]$$

$$S' = \frac{Q' - 1/Q'}{2}$$

$$T' = \frac{Q' + 1/Q'}{2}$$

$$V' = \sin\left(\frac{B \cdot u'}{A}\right)$$

$$U' = \frac{V' \cos \gamma_c + S' \sin \gamma_c}{T'}$$

$$t' = \left[\frac{H}{\sqrt{(1+U')(1-U')}} \right]^{1/B}$$

$$\chi = \pi / 2 - 2 \cdot \arctan(t')$$

$$\begin{aligned} \lambda = & \sin(2, \chi) \left(e^2 / 2 + 5e^4 / 24 + e^6 / 12 + 13e^8 / 360 \right) + \\ & \sin(4, \chi) \left(7e^4 / 48 + 29e^6 / 240 + 811e^8 / 11520 \right) + \\ & \sin(6, \chi) \left(7e^6 / 120 + 81e^8 / 1120 \right) + \\ & \sin(8, \chi) \left(4279e^8 / 161280 \right) \end{aligned}$$

$$\lambda = \lambda_0 - \arctan\left(\frac{S' \cos \gamma_c - V' \sin \gamma_c}{\cos(Bu' / A)}\right) / B$$

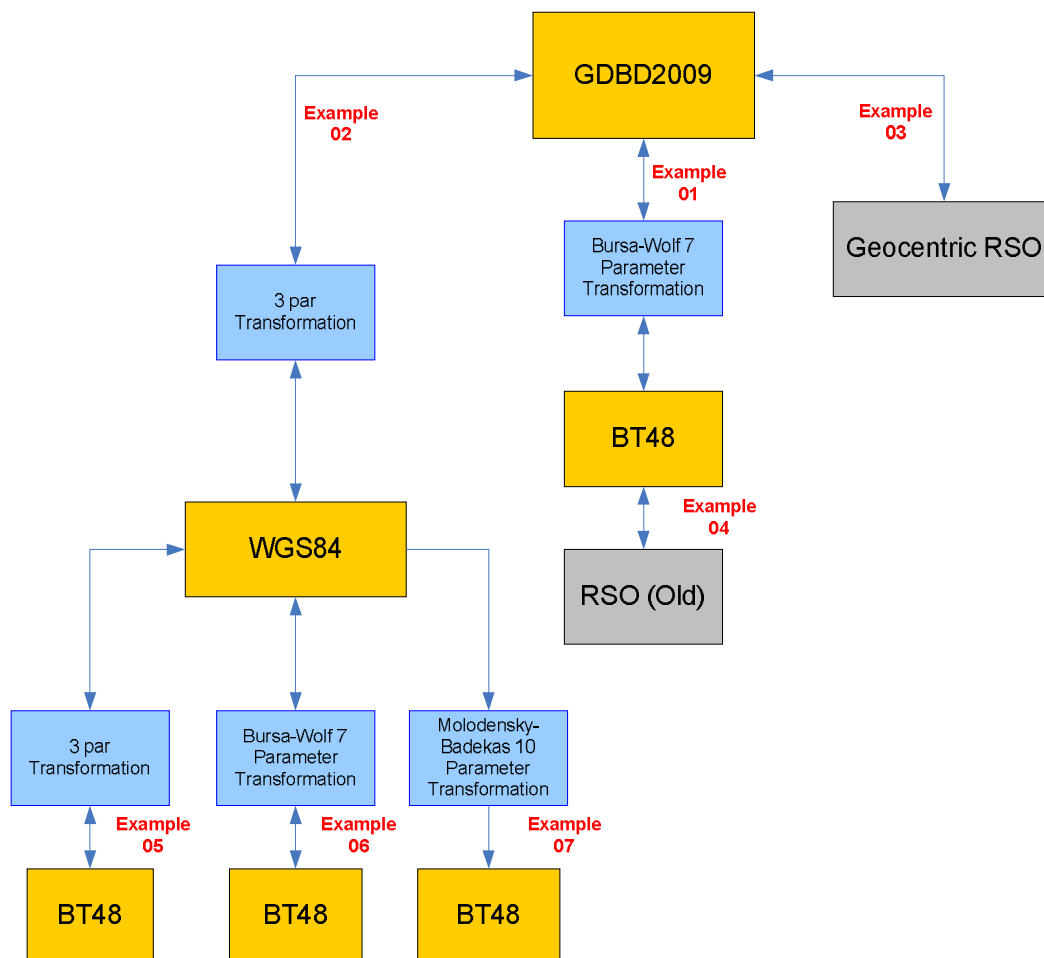
CHAPTER 4

TEST EXAMPLES

4.1 INTRODUCTION

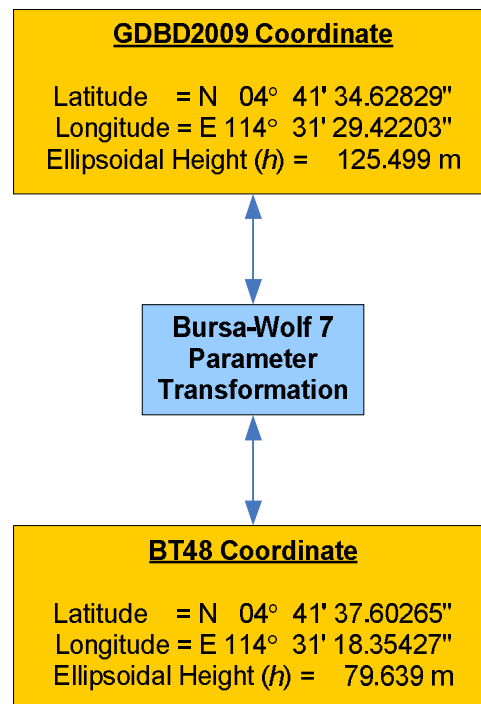
This chapter provides examples of 3D Coordinates Transformation and Map Projection Computation, which are based on the algorithms and parameters given in Chapter 3.

4.2 TRANSFORMATION FLOW



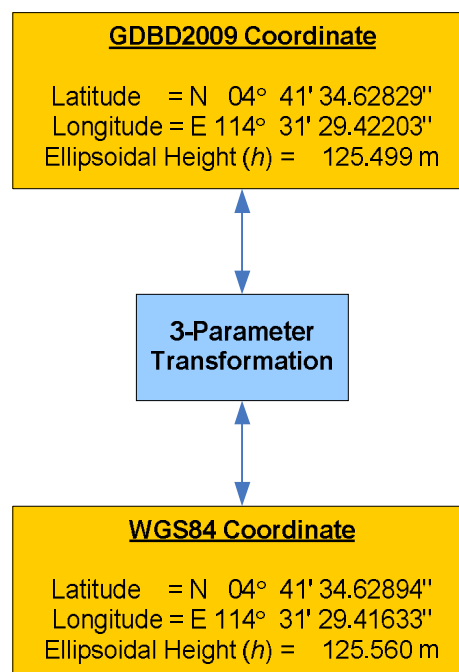
4.2.1 EXAMPLE 01

GDBD2009 TO BT48 (Bursa-Wolf 7 Parameter)



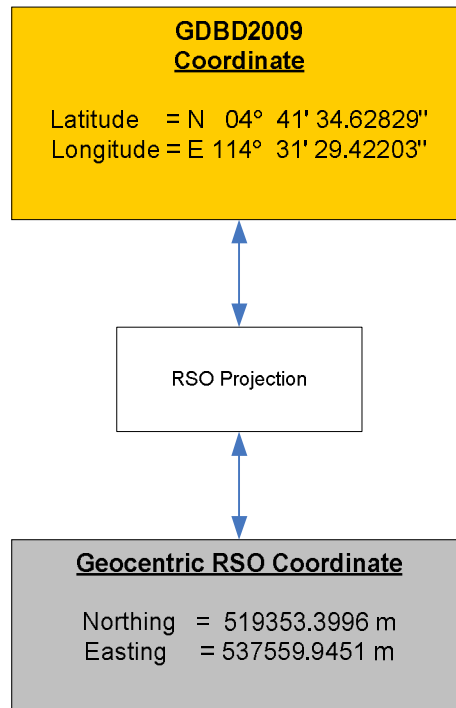
4.2.2 EXAMPLE 02

GDBD2009 TO WGS84 (3 Parameter)



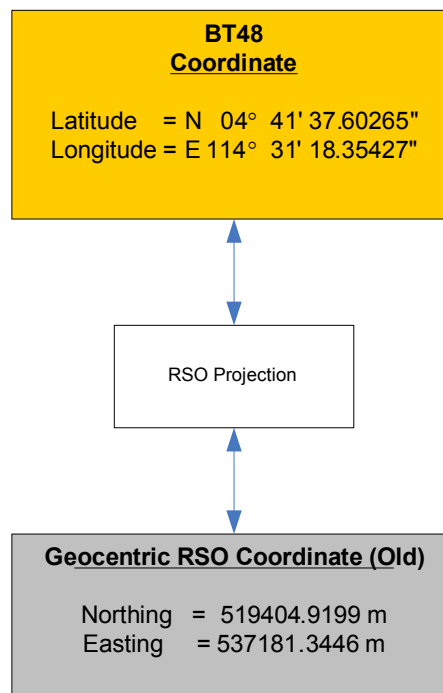
4.2.3 EXAMPLE 03

GDBD2009 TO GEOCENTRIC RSO



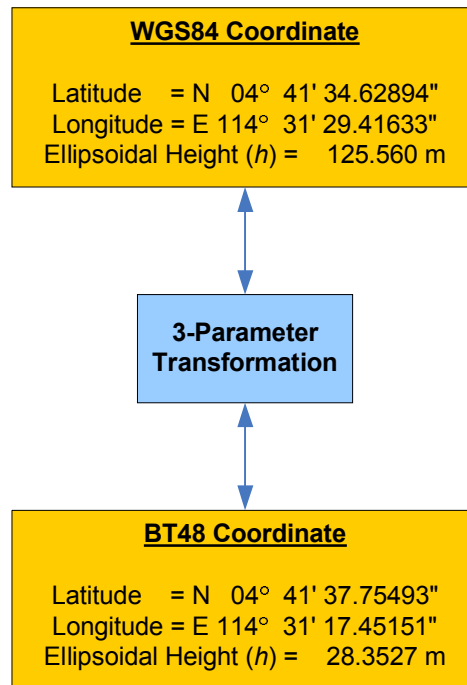
4.2.4 EXAMPLE 04

BT48 TO RSO (Old)



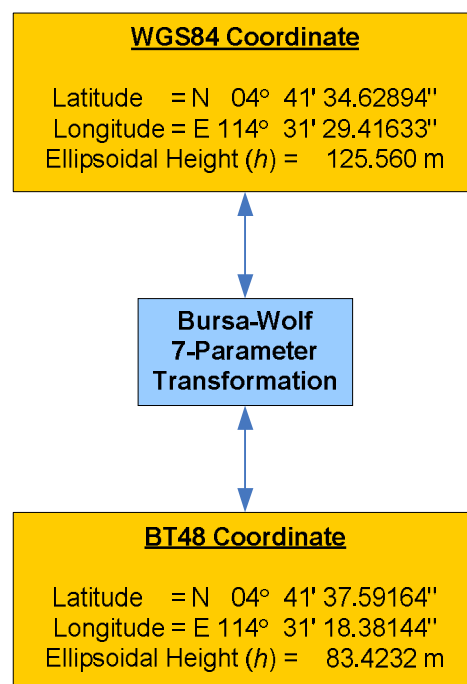
4.2.5 EXAMPLE 05

WGS84 TO BT48 (3-Parameter)



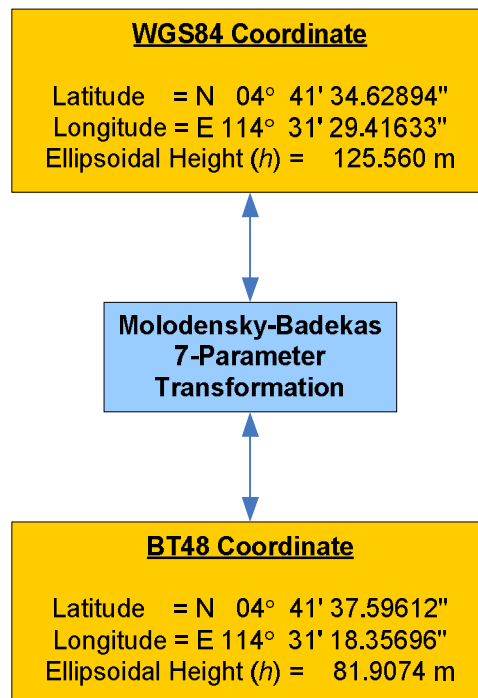
4.2.6 EXAMPLE 06

WGS84 TO BT48 (Bursa-Wolf 7-Parameter)



4.2.7 EXAMPLE 07

WGS84 TO BT48 (Molodensky-Badekas 10-Parameter)



APPENDIX A

GDBD2009 COORDINATES

No	Station Name	Latitude			Longitude			Ellipsoidal Height (m)
		°	'	"	°	'	"	
TEMB	Temburong	4	42	38.58239	115	4	22.97387	65.181
UKUR	Survey Department	4	55	31.13325	114	55	41.76840	74.740
TUTO	Tutong	4	48	24.37448	114	39	34.66312	79.978
LABI	Labi	4	24	50.02941	114	27	43.51631	75.175
MURA	Muara	5	1	59.36289	115	3	58.32242	63.822
LIAN	Sg. Liang	4	42	15.56873	114	31	29.46270	65.788
KBEL	Kuala Belait	4	34	36.72897	114	12	11.88063	62.475
LAMU	Lamunin	4	40	38.50265	114	42	43.08582	75.874
B001	Agok	4	54	10.81608	114	47	40.48858	159.552
B004	Marudi West Base	4	08	12.51469	114	20	17.37041	61.436
B007	Batu Belah	4	00	43.97811	114	29	48.80825	336.963
B008	Bedawan	4	29	31.93600	114	49	08.16156	574.667
B010	Berayong	4	46	54.19496	115	20	46.72692	772.482
B011	Bumbung Rumah	4	54	10.04586	115	30	56.29515	931.992
B012	Kala	4	01	06.10192	114	17	21.67965	138.429
B019	Lambir	4	11	56.15276	113	59	50.05005	506.891
B021	Miri	4	23	26.17784	113	59	53.25767	124.704
B025	Saeh	4	51	14.25856	114	56	01.30362	263.166
B026	Sagan A	4	48	10.09279	115	14	19.01036	103.106
B027	Sagan B	4	36	16.75911	115	01	53.09025	580.949
B032	Tunggulian	4	41	34.62829	114	31	29.42203	125.499
B033	Telingan	4	22	28.38573	114	28	16.39018	300.629
B034	Tempayan Pisang	5	00	36.27763	115	02	57.29423	190.806
B036	Bukit Timbalai	5	17	00.40526	115	11	07.16295	114.739
B038	Tunjang Pipit	4	19	37.14257	114	38	39.05205	96.369
B039	Ulu Tutong	4	16	40.52899	114	49	41.99909	422.082

TRANSFORMATION PARAMETER**GDBD2009 TO BT48 (Bursa-Wolf 7-Parameter)**

Parameter	Value	Standard Deviation
Dx	689.59370 m	± 11.31003 m
Dy	-623.84046 m	± 6.78722 m
Dz	65.93566 m	± 14.40584 m
Rx	-0.02331"	± 0.42256 "
Ry	1.17094"	± 0.27705 "
Rz	-0.80054"	± 0.37907 "
Scale	-5.88536 ppm	± 0.92598 ppm

GDBD2009 TO WGS84 (3-Parameter)

Parameter	Value	Standard Deviation
Dx	0.13513 m	± 0.07889 m
Dy	0.12670 m	± 0.07889 m
Dz	0.02497 m	± 0.07889 m

WGS84 TO BT48 (Bursa-Wolf 7-Parameter)

Parameter	Value	Standard Deviation
Dx	597.1257 m	-
Dy	-624.202 m	-
Dz	2.1991 m	-
Rx	-1.45741"	-
Ry	-0.84837"	-
Rz	1.79984"	-
Scale	-10.4358ppm	-

WGS84 TO BT48 (3-Parameter)

Parameter	Value	Standard Deviation
Dx	726.282 m	-
Dy	-703.611 m	-
Dz	48.999 m	-

WGS84 TO BT48 (Molodensky-Badekas 10-Parameter)

Parameter	Value	Standard Deviation
Dx	678.3858 m	-
Dy	-665.3742 m	-
Dz	48.2161 m	-
Rx	1.6737"	-
Ry	1.5209"	-
Rz	2.8054"	-
Scale	6.9925ppm	-
X _m	-2678448.9066 m	-
Y _m	5762777.7250 m	-
Z _m	543962.5028 m	-

Glossary

ADSL

- **Asymmetric digital subscriber line (ADSL)** is a form of DSL, a data communications technology that enables faster data transmission over copper telephone lines than a conventional voiceband modem can provide.

BT48

- **Borneo Triangulation 1948**

ECEF

- **ECEF** stands for **Earth-Centered, Earth-Fixed**, and is a Cartesian coordinate system used for GPS, and is sometimes known as a "conventional terrestrial" system. It represents positions as an X, Y, and Z coordinate. The point (0,0,0) denotes the mass center of the earth, hence the name Earth-Centered. The z-axis is defined as being parallel to the earth rotational axes, pointing towards north. The x-axis intersects the sphere of the earth at the 0° latitude, 0° longitude. This means the ECEF rotates with the earth around its z-axis. Therefore, coordinates of a point fixed on the surface of the earth do not change, hence the name earth-fixed. Conversion from a WGS84 Datum to **ECEF** can be used as an intermediate step in converting velocities to the North East Down coordinate system.

GNSS

- **Global Navigation Satellite Systems (GNSS)** is the standard generic term for satellite navigation systems that provide autonomous geo-spatial positioning with global coverage.

GPS

- The **GLOBAL POSITIONING SYSTEM (GPS)** is a global navigation satellite system (GNSS) developed by the United States Department of Defense

Helmert Transformation

- The **Helmert transformation** also called a **seven-parameter transformation** is a transformation method within a three-dimensional space. It is frequently used in geodesy to produce distortion free transformations from one datum to another using:

$$X_T = C + \mu RX$$

where

- X_T is the transformed vector
- X is the initial vector

The parameters are:

- C — translation vector. Contains the three translations along the coordinate axes
- μ — scale factor, which is unit less, and as it is usually expressed in ppm, it must be divided by 1,000,000.
- R — rotation matrix. Consists of three axes (small rotations around the coordinate axes) r_x , r_y , r_z . The rotation matrix is an orthogonal matrix. The rotation is given in radians.
- Thus, the Helmert transformation is a similarity mapping. It is a special implementation of Galilean transformation, along with affine transformations and projection transformations. However, the latter two distort the lengths of line segments.

IGS

- The **International GNSS Service (IGS)**, formerly the International GPS Service, is a voluntary federation of more than 200 worldwide agencies that pool resources and permanent GPS & GLONASS station data to generate precise GPS & GLONASS products. The IGS is committed to providing the highest quality data and products as the standard for Global Navigation Satellite Systems (GNSS) in support of Earth science research, multidisciplinary applications, and education.

ITRF

- An **International Terrestrial Reference Frame (ITRF)** is a realization of the ITRS. New ITRF solutions are produced every few years, using the latest mathematical and surveying techniques to attempt to realize the ITRS as precisely as possible. Due to experimental error, any given ITRF will differ very slightly from any other realization of the ITRF.

ITRS

- The **International Terrestrial Reference System (ITRS)** describes procedures for creating reference frames suitable for use with measurements on or near the Earth's surface. This is done in much the same way that a physical standard might be described as a set of procedures for creating a *realization* of that standard. The IERS defines a geocentric system of coordinates using the SI system of measurement.

MSL

- **Mean sea level (MSL)** is the average (mean) height of the sea, with reference to a suitable reference surface. Defining the reference level, however, involves complex measurement, and accurately determining **MSL** can prove difficult.

PPP

- The **Point-to-Point Protocol**, or PPP, is a data link protocol commonly used to establish a direct connection between two networking nodes. It can provide connection authentication, transmission encryption privacy, and compression. PPP is used over many types of physical networks including serial cable, phone line, trunk line, cellular telephone, specialized radio links, and fiber optic links.

PPPoE

- **PPPoE, Point-to-Point Protocol over Ethernet**, is a network protocol for encapsulating Point-to-Point Protocol (PPP) frames inside Ethernet frames. It is used mainly with ADSL services where individual users connect to the ADSL transceiver (modem) over Ethernet and in plain Metro Ethernet networks

RINEX

- **Receiver Independent Exchange Format (RINEX)** is data interchange format for raw satellite navigation system data. This allows the user to post-process the received data to produce a more accurate solution — usually with other data unknown to the original receiver, such as better models of the atmospheric conditions at time of measurement.

RMS

- In mathematics, the **root mean square** (abbreviated **RMS** or **rms**), also known as the **quadratic mean**, is a statistical measure of the magnitude of a varying quantity. It is especially useful when varieties are positive and negative, e.g., sinusoids.

RTK

- **Real Time Kinematic (RTK)** satellite navigation is a technique used in land survey and in hydrographic survey based on the use of carrier phase measurements of the GPS, GLONASS and/or Galileo signals where a single reference station provides the real-time corrections of even to a centimeter level of accuracy.

SLR

- **Satellite Laser Ranging (SLR)**

VLBI

- **Very Long Baseline Interferometry (VLBI)**

VRS

- A **virtual reference station** is an imaginary, unoccupied reference station which is only a few meters from the RTK user. For this position, observation data are created from the data of surrounding reference stations as though they had been observed on that position by a GPS receiver. Trimble's VRSTM networks use RTK solutions from the Trimble® RTKNet software to provide high-accuracy, real-time kinematic (RTK) GNSS positioning for wider areas. A VRS network improves productivity while at the same time reduces costs, by eliminating the need to set up a base station.

WGS84

- The **World Geodetic System** is a standard for use in cartography, geodesy, and navigation. It comprises a standard coordinate frame for the Earth, a standard spheroidal reference surface (the *datum* or *reference ellipsoid*) for raw altitude data, and a gravitational equipotential surface (the *geoid*) that defines the *nominal sea level*. WGS 84 is the reference coordinate system used by the Global Positioning System.

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